Lagrangian Modeling of Electrical Properties of the Growing Convective Boundary Layer

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ABSTRACT: Using a Lagrangian stochastic approach to the growing convective boundary layer (CBL), we have calculated the turbulent transport and drift in atmospheric electric field of such natural electrically charged substances as air ions and aerosols. The proposed numerical model is based on balance equations for the light air ions, single-charged and neutral aerosol particles which are size distributed. We have obtained the time evolution of high-altitude profiles of electric space charge, electric conductivity, vertical electric field intensity, and total vertical electric current density. We have ascertained that the spectral distribution of calculated variances of space charge density fluctuations in frequency ranges f = 0.001- 0.5 Hz is satisfied to a power dependence $S \sim f^{-\alpha}$ where $0.7 < \alpha < 2$.

INTRODUCTION

The planetary boundary layer is the lower part of the global electric circuit. This part is very thin in comparison with the distance between high-conductive Earth's surface and ionosphere that works as global capacitor plates. Nevertheless, the contribution of boundary layer electric columnar resistance to the columnar resistance of full depth of atmosphere is essential [Harrison and Bennett 2007]. The atmospheric convectively-driven boundary layer is a particular type of turbulent boundary layer forced mainly by surface heating. Electrodynamic processes within the CBL are strongly influenced by turbulent buoyant convection of ionizing and electrically charged components of air. Ionization of air is the main mechanism of electric space charge generation. The ion production rate due to ^{222}Rn and its progeny is space- and time-dependent in the CBL. Electric conductivity of air directly depends on the number density of light atmospheric ions. Therefore, conditions of ion production, their recombination, and attachment to aerosol particles have a substantial effect on electrical phenomena in the CBL. Boundary layer forced by increasing buoyancy flux deepens [Stull 1988; Garratt 1992]. It forms well-mixed part of the boundary layer and causes vertical spreading ^{222}Rn and its progeny which have been trapped within the nocturnal stable boundary layer [Vinuesa et al. 2007]. As a result, vertical profiles of mean conductivity, charge density, and electric field intensity change with changes of ion production rate.

The calculations of convection electric currents in unstable boundary layer based on second-order turbulence closure showed that turbulent transport of charge acts as a local generator which modifies the electrical structure of the ABL [Willett 1979]. In order to study the variability of electric field during morning transition, several experiments with tethered balloon were performed [Marshall et al. 1999]. It

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was found space charge by the ground to be increased after sunrise in some cases. Later low lying electric charge was spreading up to several hundred meters that vertical profiles of electric field intensity were significantly modified. Recently, the Lagrangian stochastic model has been proposed to calculate the electric field disturbances due to turbulent transport of space charge heterogeneities within the atmospheric boundary layer (ABL) [Anisimov et al. 2013a].

This paper introduces the concepts of Lagrangian stochastic modeling of electrodynamics of the CBL including morning transition period. We focus on the computation and analysis of mean and variations of vertical profiles of ionization rate, light ions, single-charged aerosol particles, electric space charge, electric field, and vertical conduction and convection currents.

MODEL DESCRIPTION

Basic equations and approximations

One of the sources of the ABL electric field is space charge originated from combined action of air ionization and microscopic separation of produced electric charges. The external electric currents maintain the global potential difference thereby they supply the quasistationary conduction current flowing through the fair weather regions of atmosphere. Within the CBL in addition to conduction current, strong vertical turbulent transport of electric charge takes place. In order to derive the air conductivity and space charge density varying with space and time, we need to know the number densities of light atmospheric ions and other charged particles. For simplicity, we take into account only light ions and aerosol particles with the unit elementary charge. The balance equations for light negative and positive atmospheric ions are

$$\begin{aligned}
\frac{\partial n_-}{\partial t}+(\boldsymbol{v}\nabla)n_- &= q(\boldsymbol{r},t)-n_-\left[\alpha n_+ +\sum_{D_a}\Big(\beta_{-0}(D_a)N_{0a}+\beta_{-+}(D_a)N_{+a}\Big)\right]+\mu_-\nabla(n_-\boldsymbol{E}),\\
\frac{\partial n_+}{\partial t}+(\boldsymbol{v}\nabla)n_+ &= q(\boldsymbol{r},t)-n_+\left[\alpha n_- +\sum_{D_a}\Big(\beta_{+0}(D_a)N_{0a}+\beta_{+-}(D_a)N_{-a}\Big)\right]-\mu_+\nabla(n_+\boldsymbol{E}).
\end{aligned} \tag{1}$$

Left hand sides (1) hold the substantial derivatives, $\boldsymbol{v}$ is the flow velocity; right hand sides hold $q(\boldsymbol{r},t)$ that is the ionization rate, recombination rate with the coefficient α, rate of ions depletion due to ion-aerosol attachment with the size-dependent coefficients β_{+0} , β_{+-} , β_{-0} , β_{-+} which correspond to adsorption of light ions by neutral aerosol particles and particles with opposite sign of charge. N_{0a} , N_{+a} , N_{-a} are the number densities of neutral, positively, and negatively charged aerosol particles with the diameter D_a correspondingly. The last terms in rhs of (1) describe the drift of ions with the mobility μ in the electric field $\boldsymbol{E}$. The equations for charged aerosol particles

$$\frac{\partial N_+}{\partial t}+(\boldsymbol{v}\nabla)N_+=\frac{n_+}{\tau_{+0}}-\frac{n_-}{\tau_{-e}},\quad \frac{\partial N_-}{\partial t}+(\boldsymbol{v}\nabla)N_-=\frac{n_-}{\tau_{-0}}-\frac{n_+}{\tau_{+e}} \tag{2}$$

hold in rhs the terms which correspond to charging of neutral particles and discharging of charged particles resulting from attachment of light ions; τ_{+0} , τ_{-0} , τ_{+e} , τ_{-e} are the lifetimes of light ions. These lifetimes are derived by replacement of summation in (1) for integration of attachment coefficients with the aerosol particle size distribution function

$$\sum_{D_a} \beta_{\pm 0}(D_a) N_{0a} = \int_{D_1}^{D_2} \beta_{\pm 0}(D) f_0(D) dD = \tau_{\pm 0}^{-1}. \tag{3}$$

The main contribution to (3) gives the size interval from D_1=10^{-6} cm to D_2=10^{-3} cm. The neutral aerosol particle size normalized distribution function is assumed to have the form [Smirnov 1992]

$$f_0(D) = \begin{cases} \left[1.5 - D_1/\widetilde{D} - 0.5\cdot(\widetilde{D}/D_2)^2\right]^{-1} \cdot N_0/\widetilde{D}, & D < \widetilde{D} = 10^{-5}\ cm \\ \left[1.5 - D_1/\widetilde{D} - 0.5\cdot(\widetilde{D}/D_2)^2\right]^{-1} \cdot (N_0/\widetilde{D})\cdot(\widetilde{D}/D)^3, & D \geq \widetilde{D}, \end{cases} \tag{4}$$

where N_0 is the number density of aerosol particles with the diameters from D_1 to D_2. The attachment coefficient for ion and charged aerosol particle is given by [Smirnov 1992]

$$\beta_{\pm}(D) = 2\pi \overline{D}_{\pm} D \left(\frac{1 + \sqrt{\pi \gamma_{\pm}(D)}}{1 + 8\overline{D}_{\pm}/D\overline{c}_{\pm}} \right), \tag{5}$$

where $\beta_+ = \beta_{+-}$, $\beta_- = \beta_{-+}$, $\beta_- = \beta_{-+}$; $\overline{D}_{\pm}$ is the mean diffusion coefficient for ion; $\overline{c}_{\pm}$ is the mean thermal velocity of ion; $\gamma_{\pm}$ is the dimensionless parameter of adsorption

$$\gamma_{\pm}(D) = \frac{e^2}{2\pi\varepsilon_0 DkT}, \tag{6}$$

where *e* is the elementary charge, *T* is the absolute temperature. For a relation between attachment coefficients one may receive the parameterization based on the results of laboratory experiments and theoretical estimations given in Hoppel [1985], Hoppel and Frick [1986]

$$\beta_{\pm 0}(D) \approx \beta_{\pm}(D) \exp\left[\frac{5.5\cdot 10^{-6}}{D}\right], \quad (D \text{ in cm}). \tag{7}$$

Substituting (5) to (7), (4) and (7) to (3), and integrating, we obtain

$$\tau_{\pm 0}^{-1} = \overline{\beta}_{\pm 0} N_0 , \tag{8}$$

where $\overline{\beta}_{\pm 0}$ is the mean attachment coefficient for light ions and neutral aerosol particles. Because of light ion attached to the particle with the diameter greater than 10^{-8} m increases its size insignificantly [Smirnov 1992], we make assumption that the charged aerosol particle size normalized distribution function has the analogous to (4) form

$$f_{\pm}(D) = \begin{cases} \left[1.5 - D_1/\widetilde{D} - 0.5\cdot(\widetilde{D}/D_2)^2\right]^{-1} \cdot N_{\pm}/\widetilde{D}, & D < \widetilde{D} = 10^{-5}\ cm \\ \left[1.5 - D_1/\widetilde{D} - 0.5\cdot(\widetilde{D}/D_2)^2\right]^{-1} \cdot (N_{\pm}/\widetilde{D})\cdot(\widetilde{D}/D)^3, & D \geq \widetilde{D}. \end{cases} \tag{9}$$

Therefore, (5) and (9) give

$$\tau_{\pm e}^{-1} = \sum_{D_a} \beta_{\pm}(D_a) N_{\mp a} = \int_{D_1}^{D_2} \beta_{\pm}(D) f_{\mp}(D) dD = \overline{\beta}_{\pm} N_{\mp} , \tag{10}$$

where $\overline{\beta}_{\pm}$ is the mean attachment coefficient for light ions and charged aerosol particles. According to Gauss's law

$$\nabla \boldsymbol{E} = \frac{e}{\varepsilon_0}\left(n_+ - n_- + N_+ - N_-\right). \tag{11}$$

To compute the mean vertical profiles of electrodynamic quantities within the CBL, we use horizontal-averaged one-dimensional form of (1), (2), and (11) thereafter. It is assumed the vertical conduction current J_z to be a high-independent above the CBL and it can be expressed as

$$J_z(t) = \frac{V_i(t)}{\int \sigma^{-1}(z,t)dz}, \tag{12}$$

where $V_i(t)$ is the potential difference between Earth's surface and ionosphere, $\sigma(z,t)$ is the air conductivity

$$\sigma = e\left(\mu_+ n_+ + \mu_- n_-\right). \tag{13}$$

From the continuity equation we have

$$J_z(t) = \left.\left(\varepsilon_0 \frac{\partial E_z}{\partial t} + \sigma E_z\right)\right|_{z=h(t)}, \tag{14}$$

where $h(t)$ is the height of the growing CBL. The expression (14) is the boundary condition from which E_z on the CBL top is extracted. The lower boundary conditions are the time series of $n_+(t)$ and $n_-(t)$ obtained from in-situ field observations (Figure1) [Anisimov et al. 2013b]. For $\sigma(z)$ above the CBL and $V_i(t)$ we use conventional parameterization [e. g. Gish 1944]. Ionization rate $q(z,t)$ is sum of rates due to cosmic rays, radiation of ground, ^{222}Rn and ^{220}Rn decaying families [Hoppel et al.1986; Bazilevskaya et al. 2008].

Lagrangian stochastic model of the CBL

The set of equations (1) and (2) in Lagrangian form does not include advective terms. It allows replace space integration by Monte-Carlo simulation the ensemble of particles in a turbulent flow given the probability density function of the random velocities advecting the particles along stochastic trajectories. Each Lagrangian trajectory or tracer is associated with a rather small finite volume of air which contains a sufficient amount of ionizing atoms such as ^{222}Rn, ^{220}Rn, and their progeny, light ions, and aerosol particles. Following the general rules for constructing of Lagrangian stochastic models to describe the dispersion of tracers in the ABL, we exploit the set of stochastic differential equations for random vertical velocity and displacement [Thomson 1987; Rodean 1996]

$$\begin{aligned} dW &= a(Z,W,t)dt + b(Z,W,t)d\varsigma , \\ dZ &= Wdt , \end{aligned} \tag{15}$$

where $a(Z, W, t)dt$ is the deterministic velocity forcing function, $b(Z, W, t)d\varsigma$ is the random velocity forcing with the quantity $d\varsigma$ being a component of a Gaussian white noise. For $a(Z, W, t)$ and $b(Z, W, t)$ are used expressions obtained in [Baerentsen and Berkowicz 1984; Thomson 1987; Luhar and Britter 1989] and which must be parameterized by the vertical profiles of the variance of the vertical velocity $\sigma_w^2(z,t)$, third moment of the vertical velocity $\overline{w}^3(z,t)$, and mean turbulent kinetic energy dissipation $\varepsilon(z,t)$. Over the past decade a lot of expressions for the vertical profiles of convective turbulence statistics are evaluated and proposed from large eddy simulation (LES), laboratory experiments

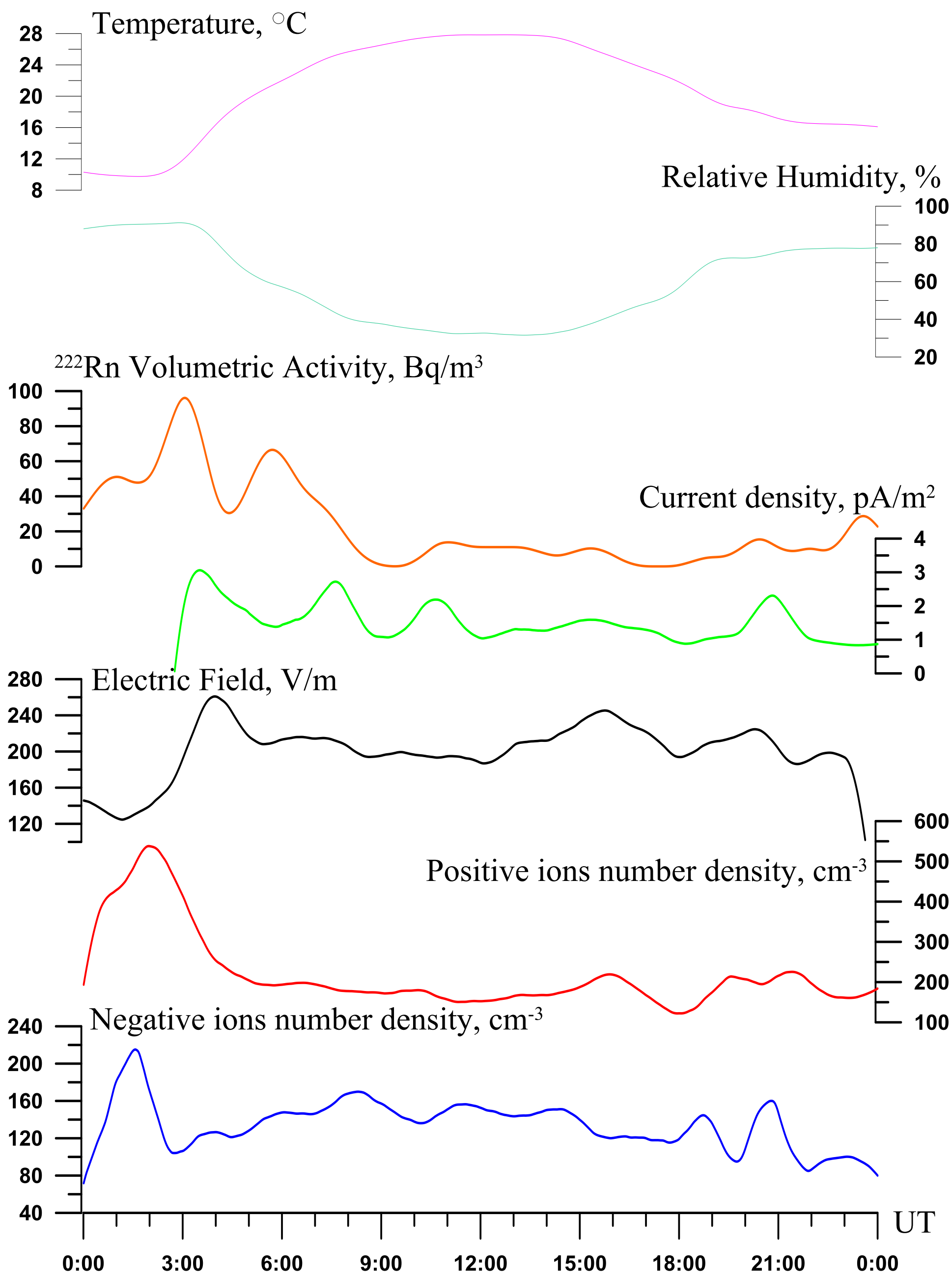

Figure 1. Example of hourly sliding averaged quantities observed on July 29, 2012 at Borok Observatory.

and natural observations [e. g. Franzese et al. 1999; Cassiani et al. 2005; Essa and Embaby 2007]. About all of these vertical profiles are the functions both of Deardorff convective velocity scale w_* and dimensionless height z/h with h being the height of mixed layer top. It implies that the turbulent statistics profiles are time-dependent since the CBL deepens. The convective velocity scale is

$$w_* = (ghH_0 / \Theta_v)^{1/3}, \tag{16}$$

where *g* is the gravity acceleration, H_0 is the kinematic turbulent heat flux at the surface, and Θ_v is the reference virtual temperature. To date several approaches are developed to describe the evolution of the CBL in terms of a few thermodynamic and energetic parameters. These approaches are based on different models of mixed layer and entrainment zone structure and dynamics. LES has played a significant role in the studies of the CBL and has given the possibility to evaluate some characteristics of the CBL evolution through comparison of LES output with the predictions of bulk or probabilistic models of the CBL [Fedorovich et al. 2004; Sorbjan 2007; Tombrou et al. 2007; Gentine et al. 2013]. The results of calculations based on an energetic model of the CBL are presented in Fig. 2.

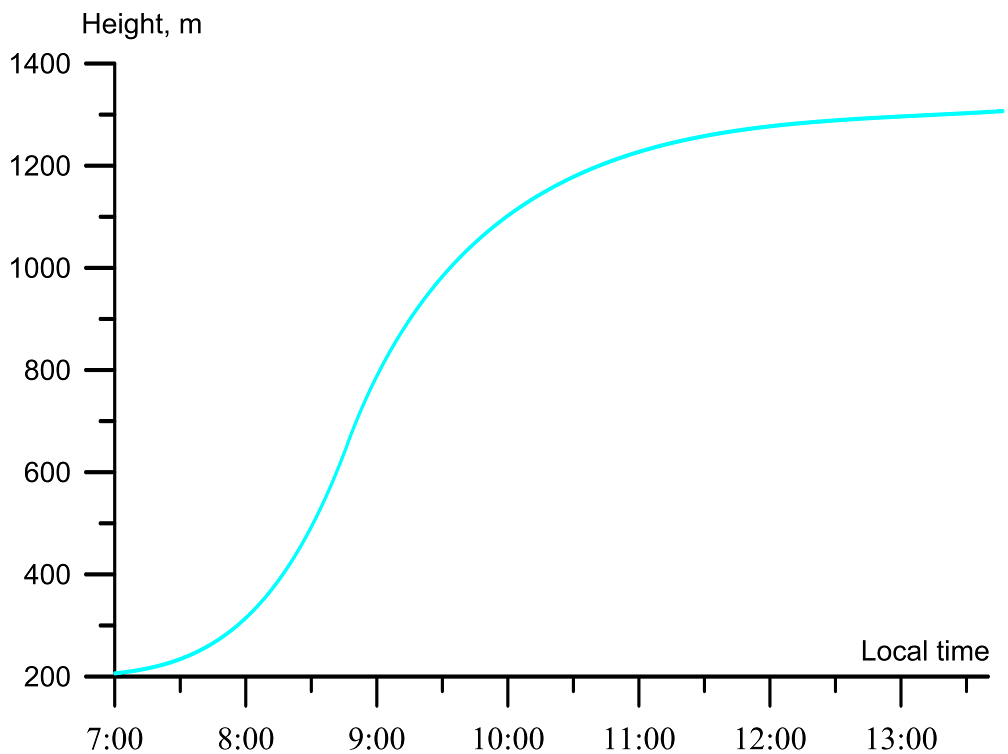

Figure 2. The CBL depth as a function of time according to the prediction of a bulk model.

Combining the Lagrangian part with the electrodynamic part by means of time decoupling of turbulent convective transport and processes of ionization, ion-ion recombination, ion-aerozol attachment, and drift of ions in self-consistence electric field, we get the vertical profiles of space charge, electric field, air conductivity, vertical conduction and convection current densities with high space-time resolution.

RESULTS AND DISCUSSION

Vertical profiles of electrodynamic quantities in the CBL

At sunrise, the solar heating causes heat transfer from ground to adjacent layer of air. The heating of low-lying air are known to be responsible for the developing of convective instability. Intensive vertical mixing tends to equalize the radioactive nuclei concentrations and, consequently, the air conductivity. The absorption of light positive ions by aerosol particles prevents their drift in electric field but does not affect on their turbulent mixing. Thus, after sunrise observed number density of light positive ions decrease, at the same time total space charge and electric field may increase [Marshall et al. 1999; Anisimov et al. 2013b].

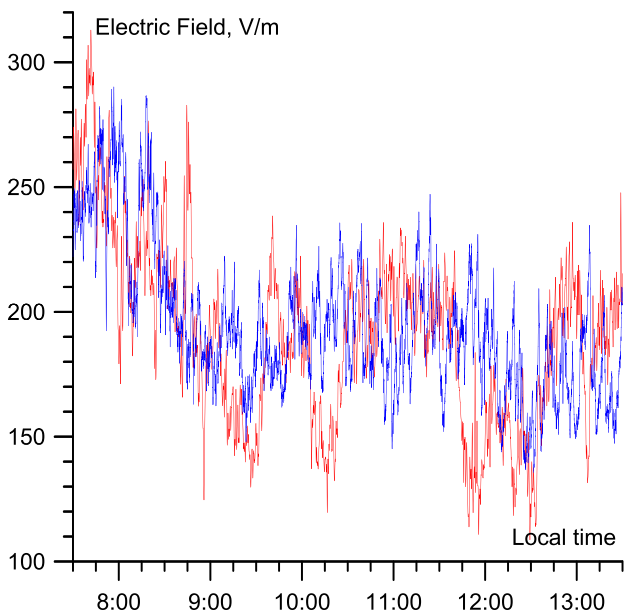

Figure 3. Example of the electric field near the surface: blue – observed on July 29, 2012 at Borok Observatory; red – calculated at the lower level of computational grid.

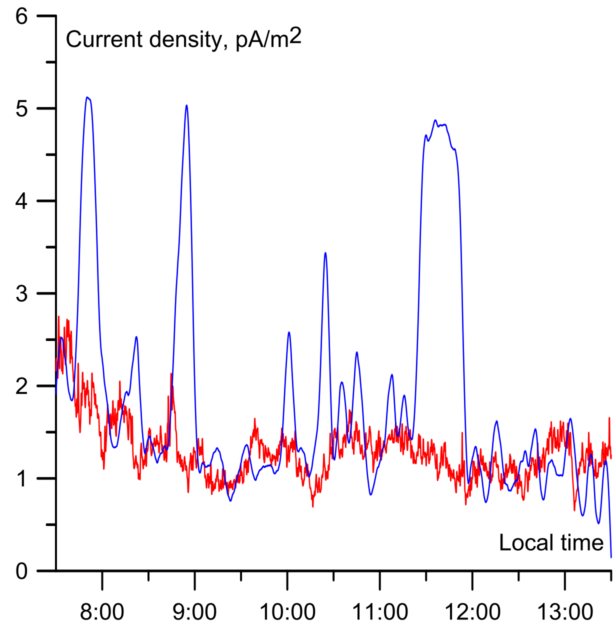

Figure 4. Example of the electric current density near the surface: blue – observed on July 29, 2012 at Borok Observatory; red – calculated at the lower level of computational grid.

Figure 3 shows the calculated electric field at the lower level of computational grid in comparison with the observed electric field. In Fig. 3 both the mean of modeled electric field and its dispersion agree with observed reasonably well. Observed with the long-wired antenna total current density variations and the calculated conduction current density at the lower level of computational grid are presented in Figure 4. We see that the total current density essentially exceed the calculated conduction current density in some intervals of time. However, its minimum is approximately equal to the calculated current density. Figure 5 and 6 show the sequence of computed electric field and conduction current density vertical profiles correspondingly. From modeled results we can conclude that adjacent to ground layer acts as the reservoir of electric charge for the convection current. When the vertical gradient of positive space charge is negative close to ground or the gradient of negative space charge is positive, the convection current is positive there and vice versa. As modeling shows values of convection current density is comparable to values of conduction current density (Figure 7).

In Figure 8 one can see the spectral density of calculated variances of space charge density fluctuations. It demonstrates self-similarity in frequency ranges f=0.001–0.5 Hz and can be approximated by a power law with the value of index from 0.7 to 2.

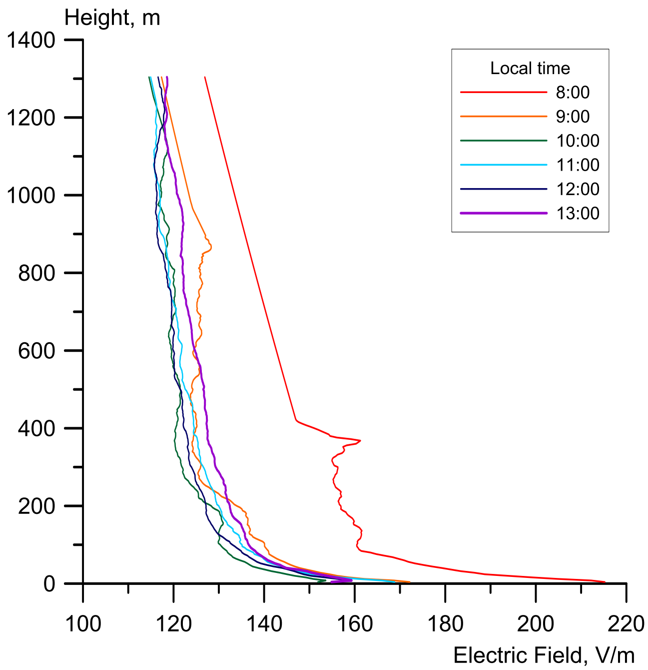

Figure 5. The 5-minutes sliding averaged vertical profiles of electric field for July 29, 2012.

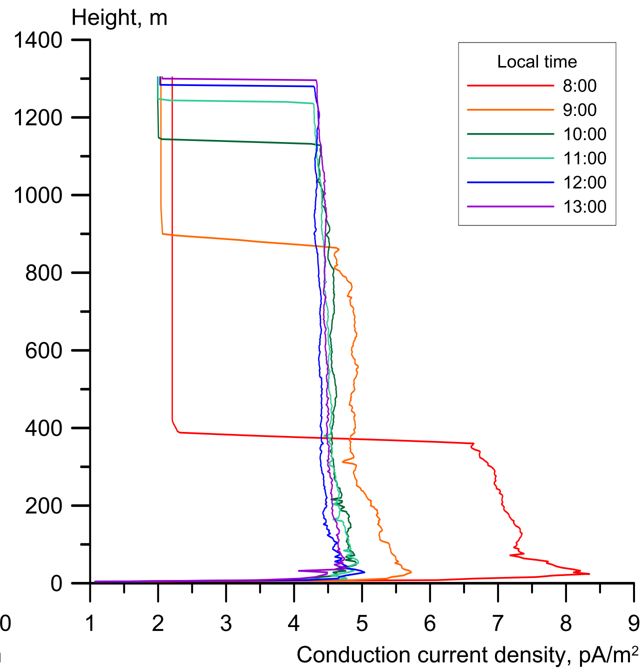

Figure 6. The 5-minutes sliding averaged vertical profiles of conduction current density for July 29, 2012.

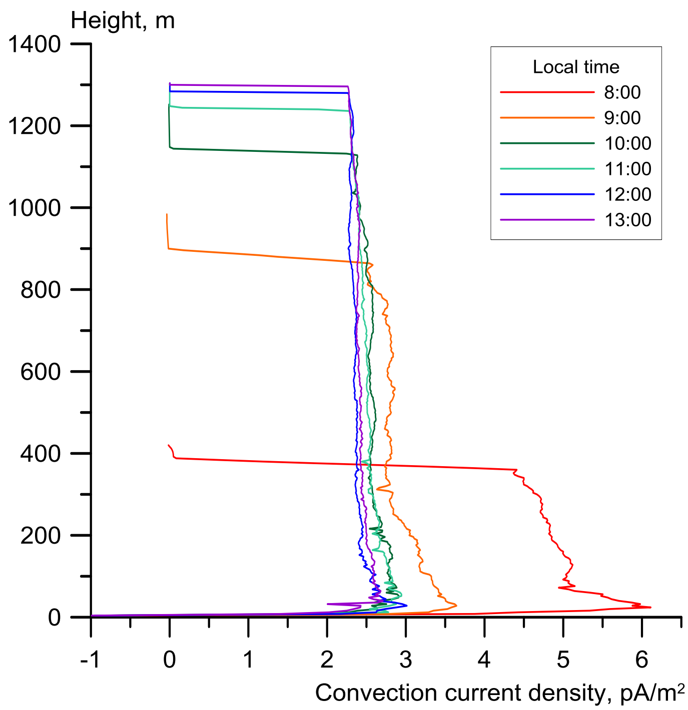

Figure 7. The 5-minutes sliding averaged vertical profiles of convection current density taken with the opposite sign that the positive value of convection current corresponds to upward direction. Calculation is performed for July 29, 2012.

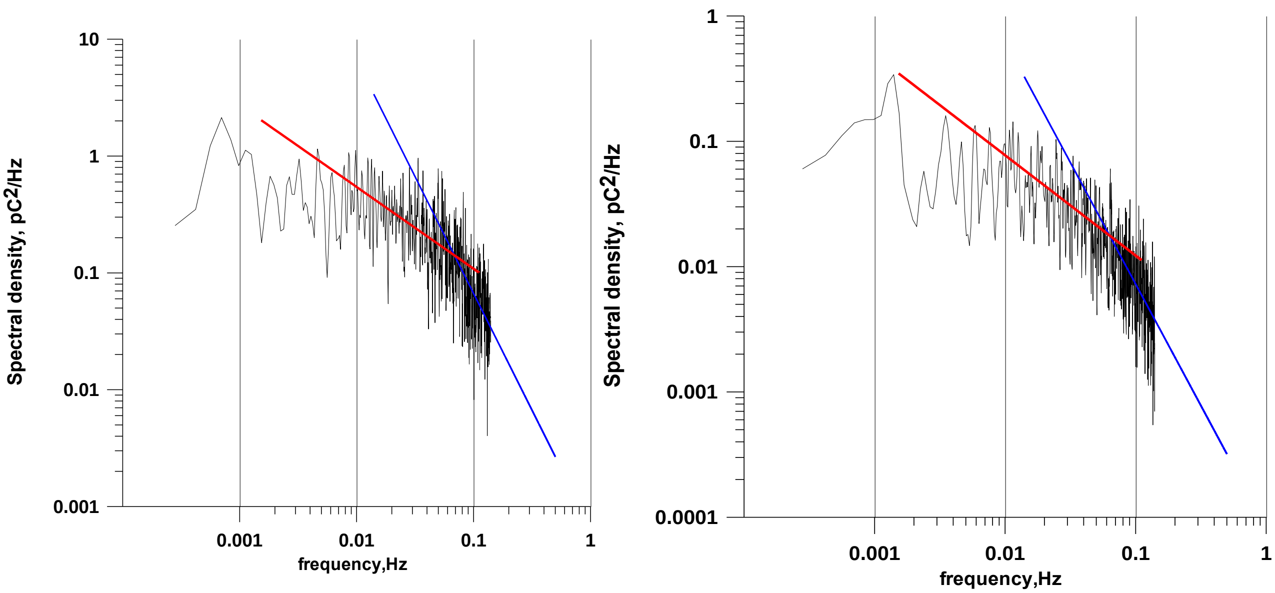

Figure 8. The spectral density of calculated variances of space charge density fluctuations for July 29, 2012, 11:30 –13:30 (LT). The figures show the results for the heights *z*=100 m (left) and *z*=1000 m (right).

CONCLUSIONS

The model we presented includes the balance equations for light atmospheric ions, neutral and charged aerosol particles, balance equations for ^{222}Rn and ^{220}Rn decaying families (not shown), Lagrangian-Eulerian part accounting for convective turbulent transport of constituents, prognostic equation for the height of mixed layer top (not shown), equations for air conductivity, current density, and electric field, dynamic boundary conditions as well.

We have seen that the Lagrangian approach allows simulate many of key properties of the CBL such as nonstationarity, nonlocality both of dispersion and processes which determine electric quantities.

It has been found the convection current to be dependent from the vertical distribution of electric space charge and may have positive or negative vertical direction.

The spectral density of calculated variances of space charge density fluctuations demonstrates self-similarity in frequency ranges f =0.001–0.5 Hz.

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