A back-of-the-envelope calculation to estimate the geometric size of thunderstorms in Saturn's atmosphere

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ABSTRACT: A terrestrial thunderstorm is a cumulonimbus cloud vertically extending from the lower troposphere up to the tropopause in which electrical discharges take place. Assuming a dipolar charge structure, it can be represented as a parallel-plate capacitor with the cloud-top (cloud-base) having a positively (negatively) charged plate. The capacitor is charged as a result of collisions between upward moving ice-crystals and the falling riming-graupel, the so-called non-inductive graupel-ice mechanism. Assuming this capacitor can only be discharged by lightning, we describe in this contribution a back-of-the-envelope calculation to compute the geometrical size of Saturnian thunderstorms. In particular, knowing that flash-rate on Earth increases linearly with the horizontal charged area of the capacitor (proportional to cloud-top area), we derive the best-fit line from flash-rate data deduced from Cassini/RPWS (Radio and Plasma Wave Science) SED (Saturn Electrostatic Discharges) records and the best-fit line from cloud-top areas as obtained from images made by amateur astronomers using ground-based optical telescopes and as published in *Sánchez-Lavega et al.* [2011]. The modeling period considered is during the initial growth stage of the Great White Spot (GWS) from 6th to 14th of December 2010. From these two linear equations of time, we derive the 'true' cloud-top area equation for Saturnian thunderstorm, which can only be obtained by adding the undetected flash-rate to the detected flash-rate ('apparent') computed from distance-normalized SEDs and measured by the same mode of the Cassini/RPWS instrument; the 'true' flash-rate is the sum of the 'apparent' and undetected flash-rates. We discuss the result of our calculation. As this is only a first step, we suggest in the outlook how other quantities describing a Saturnian thunderstorm can be estimated as well.

INTRODUCTION

The Earth and the gas planet Saturn share common features. Both planets have (a) large tilts of rotation axes w.r.t. their orbital plane, causing the planet's season to change over time; and (b) water clouds in the troposphere [*Fischer et al.*, 2008]. With this in mind, lightning on gas planets like Saturn can be understood in terms of lightning on Earth. That is, lightning storms in Saturn or Saturn Electrostatic Discharges (SEDs) [*Fischer et al.*, 2008; *Zarka et al.*, 2008] are most likely dependent on the change of the planet's season and are produced through a similar charge separation mechanism like on Earth.

The large axial tilt of Saturn may trigger effects that can couple with the change of season; yet unexplained is the way the internal heat flux is transported at the water condensation layer [*Sayanagi et al.*, 2013] near midlatitudes (as observed by instruments onboard Cassini spacecraft), depositing a sufficient CAPE, bringing instability to visible levels in the atmosphere and create eruptive, high-contrast, lightning-associated cloud features [*Desch et al.*, 2006]. When all necessary ingredients are present in forming a cloud, the cloud can become convective from its initial single cell stage to reach mature and eventually dissipation stages [*Trapp*, 2013]. If distinct, lateral and localized updrafts and downdrafts co-exist, organized convective storms can be sustained for a long period of time, including episodic SED storms. The latter coexistence of updrafts and downdrafts will allow the thunderstorm to reach the advanced stages forming a mesoscale convective system, and the dominant morphology as lightning generator could be a single/multiple-cell, or a super cell.

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For Earth's atmosphere, *Williams* [2001] has shown that flash-rate is determined by the geometric size of thunderstorms, specifically the area of the horizontal charged region, see .e.g., *Larsen and Stansbury* [1974]. In principle, if Saturnian flash-rate data are available and a series of cloud-top area measurements is available for several days from ground-based optical telescopes made by the amateur astronomers [*Fischer et al.*, 2011; *Sánchez-Lavega et al.*, 2011] when the Saturnian thunderstorm is growing towards its mature stage, then a back-of-the-envelope calculation can be performed to estimate the thunderstorm geometric size.

THEORETICAL BACKGROUND

Thunderstorm as a parallel-plate capacitor

A terrestrial thunderstorm originates from a cell that has reached a mature stage [*Trapp*, 2013]. It has a complex charge distribution, but here we assume the thunderstorm to be dipolar, a parallel-plate capacitor with the cloud-top (cloud-base) having a positively (negatively) charged plate. For convenience, we assume circularly-shaped charged layers. The capacitor is charged as a result of collisions between upward moving ice-crystals and the falling riming-graupel, the so-called non-inductive graupel-ice mechanism. We assume the capacitor can only be discharged by lightning.

Assuming the charging and discharging processes are in equilibrium, the flash-rate equation of this parallel-plate capacitor is given by [*Dahl et al.*, 2011]

$$f = \gamma \frac{j}{\Delta Q} A = \kappa A \tag{1}$$

with the constant of linear proportionality defined as

$$\kappa = \gamma \frac{j}{\Delta Q}. \tag{2}$$

That is, the flash-rate, f, depends linearly on the area of the capacitor plates, A; the lightning efficiency, γ; the charging rate, j; and the lightning charge, ΔQ.

Below, we assume that the area of the capacitor plates is proportional to or similar in size as the cloud-top area of the convective cloud.

'True', 'apparent', and unaccounted flash-rates

A flash in Saturn is identified as an SED event [*Fischer et al.*, 2011] if its intensity is above a certain threshold with respect to the background intensity. Below this threshold, a number of low intensity bursts become indistinguishable from natural fluctuation of the galactic background.

We define the 'true' flash rate,

$$f = f_1 + f_0, \tag{3}$$

as a sum of the 'apparent' or measured, and unaccounted low-intensity-SED flashrates; f_1 and f_0, respectively. Upon substituting this value of f to Eqn. (1), the number of undetected SEDs per minute can be estimated, i.e.,

$$f_1 = \kappa\, A - f_0 \approx \kappa\, A_{\mathrm{ct}} - f_0, \tag{4}$$

upon which undetected SEDs per minute, f_0, can be estimated. Here, we take the capacitor plate area to be approximately equal to the cloud-top area of the convective cloud, i.e.,

$$A \approx A_{\mathrm{ct}}. \tag{5}$$

Like the established terrestrial linear dependence, we assume the following to be valid. We can use the SED flash-rates to determine the geometric size of Saturnian thunderstorm.

DATA REDUCTION

Since 2004 the Cassini RPWS (Radio and Plasma Wave Science) instrument has so far recorded several lightning storms [*Fischer et al.*, 2008; *Zarka et al.*, 2008]. Towards the spring (autumn) equinox in northern (southern) hemisphere, the duration and strength of these SED storms that raged at 35° South, dubbed as the 'storm alley', reached their peak, which is most probably modulated by seasonal effects [*Sayanagi et al.*, 2013]. The storm alley switched to 35° North when the Great White Spot, a once per Saturnian year phenomenon, started to develop and Cassini/RPWS picked up the first outbursts on the 5th of December 2010. Around this time, amateur astronomers [*Fischer et al.*, 2011; *Sánchez-Lavega et al.*, 2011] using their optical telescopes started to see a bright spot associated to the brewing storm. Selecting only the period when the increase in the cloud-top area coincided with the increase of flash-rate, we choose the SED records covering the time interval from 6th to 14th of December 2010. After this period, cloud-top area changes non-linearly (parabolic) in time (not shown here, see Fig. 1 of *Sánchez-Lavega et al.* [2011]).

As usual, to extract the SEDs, the same algorithm as in *Fischer et al.* [2011] was used. For convenience, we consider only SED data measured from the same mode of the Cassini/RPWS instrument. This mode happens to be the most dominant mode of recording SEDs. The selected SED records have been normalized with respect to spacecraft distance to Saturn using half-orbit parameters. The details of the procedure used in normalizing the SED quantities are described elsewhere [*Pagaran and Fischer*, in prep.].

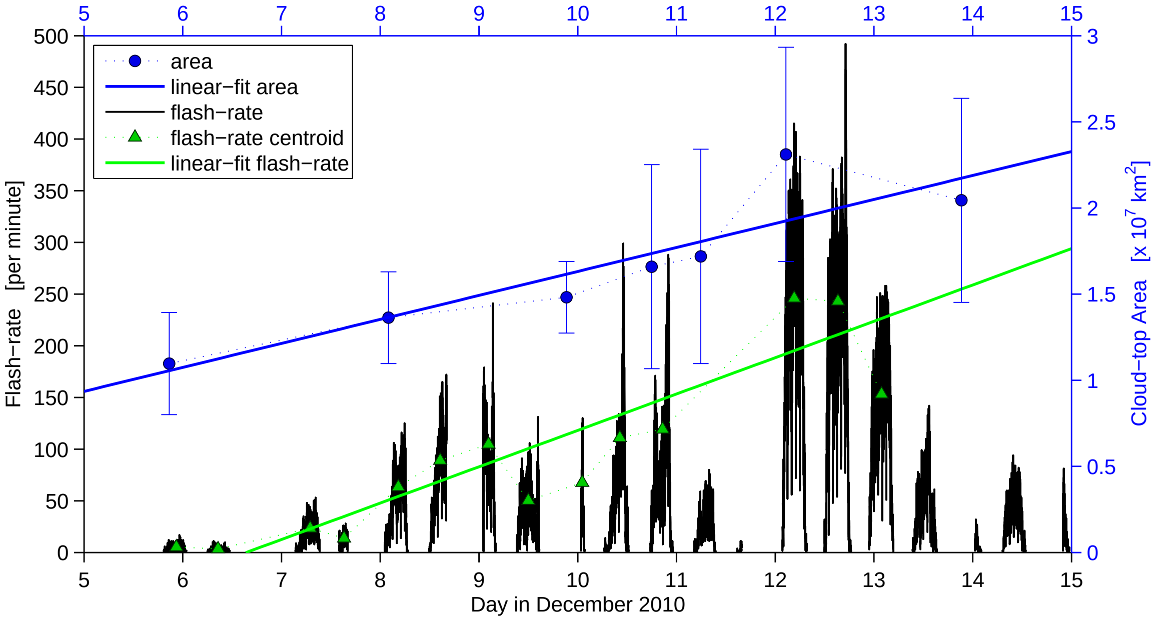

Figure 1: Plots of flash-rate (left y-axis) and cloud-top area (right y-axis) measurements during the 7-day initial growth of the Great White Spot (GWS). Flash-rates (black line) and their centroid (green filled triangle) are derived from distance-normalized SED measurements [*Pagaran and Fischer*, in prep.] that are observed from the same instrumental mode of the Cassini/RPWS instrument. Cloud-top area measurements (blue filled circles) and their r.m.s. (blue error bars) corresponding to several measurements of the storm area over each image [*Sánchez-Lavega et al.*, 2011]; Shown in both plots are the linear-fit models to the cloud-top area (blue solid line) and the centroid of flash-rates (green solid line) as a function of time. Note: only the linear part in Fig. 1 of *Sánchez-Lavega et al.* [2011] is reproduced in the figure above.

DATA ANALYSIS

In the following back-of-the-envelope calculation, we derive the best-fit line of the selected distance-normalized SED records and the best-fit line of the cloud-top area measurements published in *Sánchez-Lavega et al.* [2011]; both best-fit lines are modeled in a time interval covering the 7-day initial growth of the Great White Spot (GWS). These measurements are plotted in Fig. 1 indicated by green filled triangle for SEDs and indicated by blue filled circles (and their r.m.s. blue error bars) for cloud-top area measurements.

By using the selected distance-normalized SED records and setting the time variable t_0 to 5.8 day of the month in December 2010, we obtain the best-fit flash-rate equation (blue solid line in Fig. 1),

$$f_1 = 35.2\,t - 29.5, \tag{6}$$

(in units of SEDs per minute). Similarly by using the cloud-top area measurements [*Sánchez-Lavega et al.*, 2011] and setting the same value of t_0 as before, we obtain the best-fit cloud-top area equation (green solid line in Fig. 1),

$$A_{\mathrm{ct}} = 1393000\,t + 10553000. \tag{7}$$

Eliminating the variable t in Eqns. (6) and (7), we recover Eqn. (4) but with derived numerical values

$$f_1 = 2.5269 \cdot 10^{-5} A_{\mathrm{ct}} - 296.2, \tag{8}$$

which upon inspection, the undetected flash-rate is estimated to be given by

$$f_0 = 296.2 \approx 300. \tag{9}$$

Hence, assuming charge plate area is similar in size to cloud-top area, $A \approx A_{\mathrm{ct}}$, Eqn. (1) can be written as

$$f = f_1 + f_0 = f_1 + 300 = \kappa\, A_{\mathrm{ct}} = 2.5269 \cdot 10^{-5} A_{\mathrm{ct}}. \tag{10}$$

Or as a rule-of-thumb, up to an order-of-magnitude,

$$A_{\mathrm{ct}} = 39574\, f \approx 4 \cdot 10^4 f \tag{11}$$

in units of square kilometers.

DISCUSSION OF RESULTS

To the best of the authors' knowledge, this is the first time a back-of-the-envelope calculation is made with the flash-rate and cloud-top area of thunderstorm measurements for the gas planet Saturn. The calculation makes only sense when the SED records used are distance-normalized and only the SED records made in the same instrumental mode. Otherwise, the slope of the linear-fit model, Eqn (7), to the centroid of flash-rates data (green solid line in Fig. 1) would not be correctly determined.

Eqn. (11) is the main result of the back-of-the-envelope calculation. We have recovered an estimate for the number of undetectable SEDs, which the algorithm by *Fischer et al.* [2006] had to reject as SED events because their intensity is low, i.e., close to the galactic background fluctuation. The unaccounted flash-rate of 300 per minute that we determined from the calculation is comparable to the highest measured flash-rate of the modeling period, in particular, the flash-rate measured on 12th of December 2010. Eqn. (11) is therefore valid under the assumption that the undetectable SEDs are constant in time.

Moreover, we find the value of $4 \cdot 10^4$ square kilometers as the physical minimum horizontal area a cell in Saturn's atmosphere has to cover in order for the cell to reach a mature stage and become electrically active to generate a minimum of one SED flash per minute. We note that the synchronized linear increase of flash-rate and cloud-top area happens only in the initial stage of thunderstorm existence. A certain critical maximum area is reached when the increase of flash-rate and that of cloud-top area becomes non-linear. In this case, the cloud-top area increases quadratically (Fig. 1 of *Sánchez-Lavega et al.* [2011], after 14th of December 2010) without any further increase in flash-rate. We can say that our back-of-the-envelope calculation may be valid to a maximum cloud-top area of 22.5 million square kms. corresponding to peak flash-rates of 375 per minute, above which the number of undetected SEDs having low intensity or close to the galactic background fluctuation may no longer be constant in time. Above these peak values, the back-of-the-envelope calculation may not be valid anymore.

Judging mainly from the intensity and strength of the GWS birth, and that the measured quantities have high signal-to-noise ratio, the slope and intercept of Eqn. (11) can be said to be statistically robust even if the timeseries used in the modeling only covers one week of observation.

Together with the vertical extent of a thundercloud of 125–250 km [*Dyudina et al.*, 2013], Eqn. (11) can provide us with an order-of-magnitude knowledge with regard to the size of the Saturnian thunderstorms as well as to their most likely associated dominant convective mode [*Trapp*, 2013], i.e., whether the thunderstorm has become an ordinary single/multi-cell, or a powerful supercell.

OUTLOOK

As next steps, Eqn. (8) can be applied to SED records in each of the storm, or in several stages of each storm from the end of the year 2007 to the middle of the year 2010 to reconstruct (a) the cloud-top area and to determine (b) the growth rate of the cloud-top area. Knowing that terrestrial flash-rate increases linearly with graupel volume in vigorous updraft in upper cloud levels [*Carey and Rutledge*, 2000] and updraft surges [*Wiens et al.*, 2005; *Goodman et al.*, 2005], we can estimate (c) the Saturnian graupel volume (using estimates of the vertical extent of a thundercloud of 125–250 km [*Dyudina et al.*, 2013]) and (d) the vertical velocity from flow divergence at the top of the ascending motions as described in *Sánchez-Lavega et al.* [2011].

So far we have not given further attention to the constant of linear proportionality, κ, in the flash-rate equation given in Eqn. (2) or (11). Also, presently, we have considered the cloud-top area of thunderstorms to be proportional to the horizontal charge area of the parallel plate capacitor representing the thunderstorm. If the proportionality is known between cloud-top area and capacitor plate area, we can obtain a rule-of-thumb expression relating the lightning generator current j and the lightning charge, ΔQ. As we have assumed that the plate capacitor representing the Saturnian thunderstorm is discharged only by lightning, then by using the analytic expressions derived by *Wu and Chen* [2013], we can estimate the remnant charge, Q_0, and how it rapidly decays in time when a lightning discharge takes place. All of these aforementioned quantities depend on the geometric size of thunderstorms in Saturn's atmosphere.

Whether the estimated quantities provide new insights on the large-scale dynamics of Saturn's atmosphere is an interesting question when they are used, for example, as input parameters to convection-resolving mesoscale models, or how these quantities might compare with future in-situ measurements when the Cassini spacecraft will be sent crashing into Saturn's atmosphere as planned in September 2017.

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