Influence of the Soil and Frequency Effects to Evaluate Atmospheric Overvoltages in Overhead Transmission Lines – Part I: The Influence of the Soil in the Transmission Lines

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ABSTRACT: This paper is related to analysis of soil and frequency effects to evaluate atmospheric overvoltages in overhead transmission lines. This paper involves wave propagation and electromagnetic transients in waveguides when the soil is admitted as a dispersive medium. This consideration implies the variation of conductivity ($\sigma_{soil}$) and permittivity ($\varepsilon_{soil}$) of the soil as a function of time. These aspects are not commonly discussed by traditional literature where in general the soil is treated like a perfect electrical conductor ($\sigma_{soil} \rightarrow \infty$). However, an idealized model is not recommended to represent transmission lines when they are submitted to high frequency phenomena such as Lightning and other voltage outbreaks. In this case, the line parameters are affected by soil impedance. It is used to represent the current return along the ground.

INTRODUCTION

J. R. Carson published first studies about the soil effect in overhead transmission power lines (TL) in 1926 (Carson, 1926). In his research, he gave emphasis to soil electromagnetic properties to compute of line parameters. However, in his paper Carson had not discussed the influence of frequency on these characteristics. Carson’s model is considered a reference in these studies. However, it is associated with improper integrals. This aspect can be a problem when we make the computational implementation. Like Carson, other important research was presented regarding this subject. We can quote the works presented by M. Nakagawa (Nakagawa, 1981a and 1981b), A. Deri (Deri, 1981) and T. Noda (Noda, 2006). This last study proposed a complex plane model. Deri’s model is simpler, in computational implementation, than Carson’s equation because it has no improper integrals in its formulations.

Moreover, the soil is considered a dispersive environment and there are several methodologies that compute this characteristic. Although a dispersive medium suffers variation in its proprieties as a function of time, these methodologies corrects $\varepsilon$ and $\sigma$ in the frequency domain because it is easier to implement computationally.

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TRANSMISSION LINES PARAMETERS

The transmission lines’ parameters can be divided in two: (i) The longitudinal impedance \( Z_L \) and (ii) the transversal admittance \( Y_T \). To understand better, the following topics present each parameter and how they are considered in this paper.

**Longitudinal Impedance**

The Longitudinal Impedance computes the influence of the induced voltage and the lost in the conductor by Joule Effect. Numerically, the first part is modelled considering the idea of inductance and the second part is modelled considering the idea of electrical resistance. The resistance of \( Z_L \) has two natures: (i) Internal of the conductor \( R_{\text{internal}} \) and (ii) one resulted of the finite conductivity of the soil \( R_{\text{soil}} \). However, the inductance of \( Z_L \) has three natures: (i) Internal of the conductor \( L_{\text{internal}} \), (ii) one resulted of the consideration of the soil as a perfect conductor, the one that consider the magnetic flux concatenated by the TL \( L_{\text{external}} \) and (iii) the last one, resulted of the consideration of the finite conductivity of the soil \( L_{\text{soil}} \).

The variation of the internal impedance \( R_{\text{internal}} + j\omega L_{\text{internal}} \), known as skin effect, is calculated using Bessel’s equation of first and second species as published by H. B. Dwight (Dwight, 1918). Moreover, there are several methodologies that compute \( \sigma_{\text{soil}} \) and \( \varepsilon_{\text{soil}} \) in the parameters of transmission lines, in this paper it is known as soil’s influence. The most commonly methodology is the one proposed by J. R. Carson in 1926 (Carson, 1926). According to Carson, it is possible to consider the soil’s electromagnetic proprieties by solving (1).

\[
Z_{\text{ext}} + Z_{\text{soil}} = \frac{j\omega \mu}{2\pi} \ln \left( \frac{D_{ik}}{d_{ik}} \right) + 2 \int_0^\infty \frac{e^{-(H_i+H_k)\xi}}{\xi + \sqrt{\xi^2 + \gamma_{\text{soil}}^2}} \cos(dl_{ik}\xi) \, d\xi
\]

Where: \( \xi \) is the integration variable; \( D_{ik} = \sqrt{(H_i + H_k)^2 + dl_{ik}} \); \( dl_{ik} \) is the horizontal distance between the conductors i e k; \( \mu = \mu_0 \) is the magnetic permeability of the vacuum; \( \gamma_{\text{soil}} \) is the constant propagation of electromagnetic waves in the soil.

J. R. Carson made several suppositions to obtain his equations. M. Nakagawa (Nakagawa, 1981) reduced some suppositions made by Carson as the simplification of low and medium frequencies and the consideration of soil’s relative magnetic permeability \( (\mu_r) \). Although this last approximation is acceptable since the soil has its \( \mu_r \) approximately equal to one, Nakagawa’s equation is more precise than Carson’s. As Carson, Nakagawa stated that it is possible to consider the soil by solving (2).

\[
Z_{\text{ext}} + Z_{\text{soil}} = \frac{j\omega \mu}{2\pi} \ln \left( \frac{D_{ik}}{d_{ik}} \right) + 2 \int_0^\infty \frac{e^{-(H_i+H_k)\xi}}{\xi + \frac{\mu}{\mu_{\text{soil}}} a_1} \cos(dl_{ik}\xi) \, d\xi
\]

Where: \( \xi \) is the integration variable; \( D_{ik} = \sqrt{(H_i + H_k)^2 + dl_{ik}} \); \( dl_{ik} \) is the horizontal distance between the conductors i e k; \( \mu = \mu_0 \) is the magnetic permeability of the vacuum; \( a_1 = \sqrt{\xi^2 + \gamma_{\text{soil}}^2 - \gamma_0^2} \); \( \gamma_{\text{soil}} \) is the constant
propagation of electromagnetic waves in the soil; \( \gamma_0 \) is the constant propagation of the vacuum.

Although Carson and Nakagawa presented an amazing evolution from the point of view of electromagnetic waves propagation in TL, there was a problem with its equations; it does not have an analytical solution. To solve these problems we need to use a computational algorithm that wastes lots of processing effort. By the year of 1981, A. Deri (Deri, 1981) proposed an alternative solution for this problem. She introduced a complex depth on Lord Kelvin’s method of image. According to Deri, the soil’s conductivity can be inserted by adding a depth \((p)\) in the system equivalent proposed by Lord Kelvin, as shown on Figure 1, where \( p = 1/\sqrt{j\omega\mu_0(\sigma_{soil} + j\omega\epsilon_{soil})} \).

Even though the simulations indicate that Deri’s approximation has results as good as the solution of Carson’s Equations, T. Noda (Noda, 2006) published a paper in 2006 with a double logarithm approximation that considers two complex plans and considers the longitudinal distance of the conductors. T. Noda’s complex ground is shown on Figure 2, where

\[
\alpha = \begin{cases} 
0.07360 & (\theta \leq 50,45^\circ), \\
0.00247 \cdot \theta - 0.05127(\theta \geq 50,45^\circ) & \text{otherwise},
\end{cases} \\
\beta = 1 - A e^{\frac{1 - Aa}{1 - A}}.
\]

**Transversal Admitance**

The transversal admitance computes the capacitive effect. To see the influence of the soil in the capacitive effect, there are two sets of results: (i) Soil modelled as an ideal soil \((\sigma_{soil} \rightarrow \infty)\) and (ii) with the consideration of the finite value of \(\sigma\).

To “correct” the LT’s capacitance there are several methodologies in literature. In this paper only two are used: (i) M. Nakagawa (Nakagawa, 1981a) and (ii) F. M. Tesche (Tesche, 1992).
As in his formulations for the Longitudinal Impedance, Nakagawa also presented a paper to calculate the influence of the soil on the transversal admittance of TL’s. According to Nakagawa, to correct the capacitance effect, it is only necessary to solve the following equations, (3), (4) and (5):

\[ [C] = [P]^{-1} \]  \hspace{1cm} (3)

\[ P_{ik} = \frac{1}{2\pi \varepsilon_0} \left[ \ln \left( \frac{D'_{ik}}{d_{ik}} \right) + M + jN \right] \]  \hspace{1cm} (4)

\[ M + jN = 2 \int_0^\infty \frac{(l + \frac{\mu_1}{\mu_0} a_1) e^{-\gamma l}}{(l + \frac{\mu_1}{\mu_0} a_1)(l/\tau^2 + \frac{\mu_1}{\mu_2} a_1)} \cos(yl) \, dl \]  \hspace{1cm} (5)

Where: \( a_1 = \sqrt{l^2 + \gamma_1^2} - \gamma_0^2 \); \( \gamma_0^2 = -\omega^2 \mu_0 \varepsilon_0 \); \( \gamma_1^2 = j\omega \mu_1 (j\omega \varepsilon_1 + \sigma) \); \( \tau^2 = \frac{\gamma_1^2}{\gamma_1^2} \); \( e \) and \( y \) is the horizontal distance between the conductors i.e. k. And since this solution needs to solve an improper integral that has no analytical solution, F. M. Tesche came up with an asymptotical that consumes less computational effort (Tesche, 1992). Tesche proposed that it is possible to obtain the capacitance correction by using longitudinal parameters, as presented in (6).

\[ [Y_{solo}] = [(\gamma_{solo})]^2[Z_{solo}]^{-1} \]  \hspace{1cm} (6)

**CONDUCTIVITY AND PERMITTIVITY OF THE SOIL**

It is known that the soil is a dispersive environment. Due to this fact, it is necessary to evaluate this influence on the electromagnetic parameters of the TL. To compute the conductivity and permittivity variation on the calculus of the TL’s parameters, since these parameters are only existents on a frequency domain, the following methodologies are applied to correct them:

i. **H. Scott** (Scott, 1967)

H. Scott developed the first work addressed in this paper on 1967, where he estimated the conductivity and permittivity of the soil for a range of 100 Hz to 1 MHz. Using advanced techniques of statistics and inserting signals of several frequencies (in the laboratory), he developed the equations (7) and (8) that correct the conductivity and permittivity of the soil.

\[ K = 0.028 + 1.098K_{100} - 0.068F + 0.036K_{100}^2 - 0.046FK_{100} + 0.018 \]  \hspace{1cm} (7)

\[ D = 5.491 + 0.946K_{100} - 1.097F + 0.069K_{100}^2 - 0.114FK_{100} + 0.067 \]  \hspace{1cm} (8)

Where: \( F \) is the logarithm of the frequency (on base 10); \( K_{100} \) is the logarithm of the conductivity measured at 100 Hz (mS/m); \( K \) is the logarithm of the corrected conductivity and \( D \) is the logarithm of the dielectric constant corrected.

ii. **K. Smith and L. Longmire** (Smith, 1975)

Based on Scott’s measurements L. Longmire and K. Smith developed expressions to correct \( \sigma \) and \( \varepsilon \) for a range of 100 Hz to 1 MHz. They based their formulation on the idea that each element of the soil can be
modelled as electrical circuits of differential resistors and capacitors. The results of each circuit are the correction of the conductivity and permittivity of the soil written in equations (9) and (10).

\[ \varepsilon_r = \varepsilon_\infty + \sum_{n=1}^{N} \frac{a_n}{1 + \left(\frac{f}{f_n}\right)^2} \]  
(9)

\[ \sigma = \sigma_l + 2\pi\varepsilon_0 \sum_{n=1}^{N} \frac{a_nf_n}{1 + \left(\frac{f}{f_n}\right)^2} \]  
(10)

Where: \( \sigma_l \) is the conductivity of soil measured on low frequency; \( f_n = \left(\frac{P}{10}\right)^{1.28} \times 10^{n-1} \) Hz, \( a_n \) is given on Table 1 and \( P \) is the water percentage of the soil sample.

Table 1 - Coefficient for universal soil proposed by L. Longmire e K. Smith

<table>
<thead>
<tr>
<th>n</th>
<th>a_n</th>
<th>N</th>
<th>a_n</th>
<th>n</th>
<th>a_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,4\times10^6</td>
<td>6</td>
<td>1,33\times10^2</td>
<td>11</td>
<td>9,8\times10^{-1}</td>
</tr>
<tr>
<td>2</td>
<td>2,74\times10^5</td>
<td>7</td>
<td>2,72\times10</td>
<td>12</td>
<td>3,92\times10^{-1}</td>
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<tr>
<td>3</td>
<td>2,58\times10^4</td>
<td>8</td>
<td>1,25\times10</td>
<td>13</td>
<td>1,73\times10^{-1}</td>
</tr>
<tr>
<td>4</td>
<td>3,38\times10^3</td>
<td>9</td>
<td>4,8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5,26\times10^2</td>
<td>10</td>
<td>2,17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

iii. S. Visacro and C. Portela (Visacro, 1987)

S. Visacro and C. Portela based on Longmire and Scott, present another methodology to correct conductivity and permittivity of the soil. To obtain their expressions they use samples collected from Brazil, more specifically from the state of Minas Gerais. According to Visacro, using the equations (11) and (12) it is possible to correct the soil’s parameters. The range of their work is also between 100 Hz and 1 MHz.

\[ \rho = \rho_0 \left(\frac{100}{f}\right)^{0.072} \]  
(11)

\[ \varepsilon_r = 2,34\times10^6 (\rho_0)^{-0.535} f^{-0.597} \]  
(12)

iv. C. Portela (Portela, 1999)

Yet with the intention of studying the soil front of phenomena of high frequency (as lightning), C. Portela developed a methodology that covers a higher range of frequency (between 100 Hz and 1 MHz). According to Portela, the solution of equation (13) corrects soil’s parameters.

\[ \sigma + j\omega\varepsilon = \sigma_0 + \Delta i \left[\cotg\left(\frac{\pi}{2}\alpha_m\right) \pm j\left(\frac{\omega}{2\pi\times10^6}\right)^{\alpha_m}\right] \]  
(13)

Where the parameters \( \Delta i \) and \( \alpha_m \) used in this paper are the reasonable safety proposed by Portela (\( \Delta i = 11,71 \) (mS/m) e \( \alpha_m = 0,706 \)).
S. Visacro and R. Alípio (Visacro, 2013)

S. Visacro and R. Alípio developed a methodology that correct the parameters $\sigma$ and $\varepsilon$ of the soil based on experimental results. Their equation is valid on a range between 100 Hz and 4 MHz. R. Alípio has interesting information that the values of the information were obtained in locus. Equations (14) and (15) are the ones proposed by (Visacro, 2013).

$$\rho = \rho_0 \{1 + [1.2 \times 10^{-6} \rho_0^{0.73}] \cdot [(f - 100)^{0.65}] \}^{-1}$$  \hspace{1cm} (14)

$$\varepsilon_r = 7.6 \times 10^{-3} f^{-0.4} + 1.3$$  \hspace{1cm} (15)

**RESULTS**

To compare the influence of each methodology the system printed on the Figure 3 and the Table 2 defines the geometric distribution of the TL. Since there are 5 conductors the matrix of each parameter is a $5 \times 5$ matrix. To evaluate more easily the soil’s influence the results are presented with a transposed system, grounded-wire eliminated and in the modal domain, using the matrix of transformation of Fourtescue. Due to the limitation of pages, the results are presented only on the homopolar mode.

![Figure 3 - System Geometric to be simulated.](image)

<table>
<thead>
<tr>
<th>LT Characteristics 345 kV – Cemig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operational Voltage</td>
</tr>
<tr>
<td>Number of conductors/phase</td>
</tr>
<tr>
<td>Type of conductors phase</td>
</tr>
<tr>
<td>Code of conductors phase</td>
</tr>
<tr>
<td>Number of cable grounded-wires</td>
</tr>
<tr>
<td>Type of cables grounded-wires</td>
</tr>
<tr>
<td>Code of conductors grounded-wires</td>
</tr>
<tr>
<td>Distance among subconductors</td>
</tr>
<tr>
<td>Height phase A</td>
</tr>
<tr>
<td>Height phase B</td>
</tr>
<tr>
<td>Height phase C</td>
</tr>
<tr>
<td>Height grounded-wires</td>
</tr>
<tr>
<td>Diameter of cables of each phase</td>
</tr>
<tr>
<td>Diameter of cables of each grounded-wires</td>
</tr>
<tr>
<td>M</td>
</tr>
<tr>
<td>N</td>
</tr>
</tbody>
</table>

**Longitudinal Resistance**

The Figure 4 shows the influence of each methodology proposed to compute $\sigma$ and $\varepsilon$ of the soil on the total resistance ($R_{\text{TOTAL}} = R_{\text{soil}} + R_{\text{internal}}$). The same graphics expose on a same plot the influence for soil’s with higher (100 $\Omega$.m) or lower (10,000 $\Omega$.m) resistivity. As shown in Figure 4, for soil’s with higher resistivity the methodologies have more variation. It is expected because the soil’s influence is more
accentuated since the conductivity decreases. Although in the higher spectrum of frequency and soil’s with less conductivity there are more numerical differences, the methodologies of Carson, Deri, Taku Noda and Nakagawa present almost the same values independently of the frequency and conductivity. This interpretation is important because it carries the information that it does not matter which methodology is used because they all present the same value. Figure 5 corrects the variation of $\sigma$ and $\varepsilon$ with the frequency. As in presented in Figure 4 the biggest variation among the methodologies are for soil’s with lower conductivity and in the higher spectrum of frequency. Despite the similarity with the Figure 4 there are several differences. For example, the methodologies of Alípio, Longmire, Portela, Scott and Visacro show different results of the non-consideration of the variation with the frequency and difference among them.

Figure 4 – Total Longitudinal Resistance. All methodologies that compute conductivity and permittivity of the soil in the final result. Values of $\sigma$ and $\varepsilon$ invariants with the frequency. Homopolar mode (Transformation Matrix of Fourtescue).

Figure 5 – Total Longitudinal Resistance. All methodologies that correct conductivity and permittivity of the soil in the final result. Values of $\sigma$ and $\varepsilon$ computed according Carson. Homopolar mode (Transformation Matrix of Fourtescue).

**Longitudinal Reactance**

The Figures 6 and 7 show the influence of each methodology proposed to compute $\sigma$ and $\varepsilon$ of the soil on the total reactance ($\omega L_{\text{TOTAL}} = \omega L_{\text{soil}} + \omega L_{\text{internal}} + \omega L_{\text{external}}$). The Figure 6 is for soils of higher resistivity and the Figure 7 for soils of lower resistivity. According to Figures 6 and 7 there are almost no differences among the methodologies. It occurs because for the longitudinal reactance the most significant parameter is the external. Since this parameter does not suffer from any influence of the soil the conductivity and permittivity of the soil are not as important as it is for the longitudinal resistance. The Figure 8 shows that even for the soils with higher conductivity and lower conductivity they have approximately the same numerical values. This information is important because the soil’s parameters influence the longitudinal resistance more sharply.
Transversal Capacitance

Another interesting conclusion that is obtained in this paper is about the percentage of variation of the transversal capacitance considering the capacitance corrected as base (variation \(\% = \frac{C_{\sigma-\infty}}{C_{\text{corrected}}} \times 100\)). Figures 9 and 10 compare the influence of each methodology that corrects the capacitance of the TL. According to Figure 9 the consideration of the soil has a maximum impact of 25% compared with the non-consideration of the soil at all. However, as shown on Figure 10, for soils with resistivity less than 100 \(\Omega\cdot m\) it is not necessary to calculate the influence of the soil. When considering the variation of \(\sigma\) and \(\varepsilon\) of the soil the percentage difference decreases to less than 12% (worst case), according to Figure 11.

Figure 6 – Total Longitudinal Reactance. All methodologies that computes conductivity and permittivity of the soil in the result. Values of \(\sigma\) and \(\varepsilon\) invariants with the frequency, soil with higher resistivity. Homopolar mode (Transformation Matrix of Fourtescue).

Figure 7 – Total Longitudinal Reactance. All methodologies that computes conductivity and permittivity of the soil in the result. Values of \(\sigma\) and \(\varepsilon\) invariants with the frequency, soil with lower resistivity. Homopolar mode (Transformation Matrix of Fourtescue).

Figure 8 – Total Longitudinal Reactance. Carson’s Equation to compute conductivity and permittivity of the soil in the result. Values of \(\sigma\) and \(\varepsilon\) invariants with the frequency, soil with lower resistivity. Homopolar mode

Figure 9 – Variation percentage of the capacitance [\%]. All methodologies that computes conductivity and permittivity of the soil in the result. Values of \(\sigma\) and \(\varepsilon\) invariants with the frequency, soil with 100 \(\Omega\cdot m\) of resistivity. Homopolar
CONCLUSION

The results printed in this paper show that the parameters that suffer from more influence on the soil is the longitudinal resistance. It is more marked on soils with higher resistivity and in the upper spectrum of frequency. For the Longitudinal parameters, according to the results, the methodologies of Deri and Taku Noda have almost the same values of using any other methodology. Since Deri and Taku are asymptotical approximation and does not require numerical solution, they consume less computational effort to give the same values. Although there are influences on the transversal parameters, the consideration of the finite conductivity may be unnecessary on the final result, especially because there are other simplifications made along the electromagnetic model. The values obtained with the consideration of the conductivity consume a lot of computational effort and may be a waste of time. For the transversal parameters, the worst case (using the methodology that presents the higher variation, upper spectrum of frequency, resistivity of 10.000 Ω.m) presents a variation percentage maximum of approximately 23% for the soil with parameters invariant with the frequency. When the variation is computed it decreases to approximately 11% for the second worst case. Thus, it is conclusive that the non-consideration of the soil is a valid approximation for this parameter considering the computational effort needed to obtain the final value. Considering further that Visacro and Alfípio´s methodology is the one with less laboratorial intrinsic errors, because his work was obtained in locus, the percentage of variation decreases even more (approximately to 2.5%), strengthening the theory of the unnecessary consideration of this influence on the transversal capacitance. Another consideration that must be made is that the transversal susceptance is given by ωC; thus the variation of the capacitance is masked because it happens in higher frequencies, and ω increases the value independently of the small variation of C.

More conclusions and sensibility analyses are presented on the second part of this paper, where the influence on the overvoltage on TL is considered for a specific case.
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REFERENCES


