The influence of thunderstorm's lightnings on the global electrical circuit

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ABSTRACT: The influence of thunderstorm's lightning of the different types on the global electrical circuit is considered by the nonstationary theory of the global circuit. Variations of the electrical potential of the ionosphere and the strength of electrical field near the ground due to the thunderstorm's lightning are calculated.

1. INTRODUCTION

Variations of electrical potential of the ionosphere and the electrical field strength near the ground due to the thunderstorm's lightning are calculated by nonstationary theory of the global electrical circuit. Variations of the ionospheric potential due to intracloud lightning and positive cloud-to-ground lightnings are negative. Negative cloud-to-ground lightnings produce the positive variations of ionospheric potential. Estimates show that the input of negative cloud-to-ground lightnings in the variations of the electrical field strength is small ($\leq 2\%$). This estimations show that the global electric circuit are maintained by quasistationary stage of the evolution of thunderstorms, which characterized making of thunderstorm electrical charges.

2. MATHEMATICAL MODEL

The governing equation for the nonstationary global electric circuit in the spherical coordinates (r, θ, ϕ) with the region in the Earth's center, taking in to account thunderstorm generators is represented in the form [Morozov, 2005]:

$$\left(\frac{1}{4\pi}\frac{\partial}{\partial t} + \lambda(r)\right) \left[\frac{1}{r^{2}}\frac{\partial}{\partial t}\left(r^{2}\frac{\partial\varphi}{\partial r}\right) + \frac{1}{r^{2}\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right) + \frac{1}{r^{2}\sin^{2}\theta}\frac{\partial^{2}\varphi}{\partial\phi^{2}}\right] + \frac{\partial\lambda}{\partial r}\frac{\partial\varphi}{\partial r} = \sum_{S=1}^{N}div\vec{j}_{S} = \\
= -\sum_{S=1}^{N} \left[\frac{I_{CS}(t)}{r^{2}} + \frac{I_{CC}^{S}(t)}{r^{2}}\right] \left[\delta(r - r_{S0}) - \delta(r - r_{S1})\right] + \frac{I_{Cg}^{S}(t)}{r^{2}} \left[\delta(r - R) - \delta(r - r_{Si})\right], \qquad i = 0.1.$$

$$\vec{E} = -grad\varphi, \qquad \lambda(r) = \lambda_{0}e^{\alpha(r - R)}, \qquad \alpha = (0, 2 - 0, 3)km^{-1}$$

where Φ is the electrical potential, \vec{E} is the electrical field strength, $\lambda(r)$ – the electrical conductivity of the atmosphere, $I_{CS}(t)$ is the electrification current supplied by sth thunderstorm generator, $I_{CS}^S(t)$ is the current, related to cloud-to-gloud discharges of sth thunderstorm generator, $I_{CS}^S(t)$ is the current, related to cloud-to-ground discharges of sth thunderstorm generator, $\delta(u)$ is the delta function, Φ_S and Θ_S are the angular coordinates of the sth thunderstorm source, N is the number of thunderstorm generators operating over the entire globe at a given instant, r_{S0} and r_{S1} are the radial distances corresponding to the positive and negative charges of a thunderstorm respectively ($r_{S0} > r_{S1}$), λ_0 is the electrical conductivity near the Earth's surface and R is the Earth's radius, i is the index corresponds to the discharge of a negative lower charge in to the ground at i=1 and to the discharge of positive upper charge in to the ground at i=0.

The current $I_{CS}(t)$ is responsible for generation of electric charges in a thundercloud due to collision between cloud and precipitation particles. This current is the sum of the electric currents of

positive particles moving up ward together with the convective airflow and of negative precipitation particles moving down ward unde the action of gravity [Imyanitov, 1981]. This current can be determined in the following way:

$$I_{CS}(t) = I_{CSO}\left(1 - e^{-\frac{t - t_S}{\tau_S}}\right) \theta(t - t_S)$$
 (2)

where t_S is the time of beginning of the sth thunderstorm generator's operation, τ_S is the relaxation time dependent on the particle collisions in a thundercloud and $\theta(t-t_S)$ is Heaviside function.

The currents $I_{\it CC}^{\it S}(t)$ and $I_{\it Cg}^{\it S}(t)$ are represented in the form

$$I_{CC}^{S}(t) = -\sum_{n=1}^{N_{SC}} \Delta Q_{SC} \delta(t - nT_{S}^{C})$$
 (3)

$$I_{Cg}^{S}(t) = -\sum_{n=1}^{N_{Sg}} \Delta Q_{Sg} \delta(t - nT_{S}^{g})$$
 (4)

where ΔQ_{SC} and ΔQ_{Sg} are the quantities of charges neutralized during a discharge, T_S^C and T_S^g are the time intervals between discharges in the sth thunderstorm generator, N_{SC} , N_{Sg} are the numbers of the cloud-to-gloud discharges and of the cloud-to-ground discharges in sth thunderstorm generator.

Using (1), (2), (3), (4) Morozov [2005] was obtained the following representation for ionospheric potential $\varphi_{\infty}(t)$:

$$\varphi_{\infty}(t) = \varphi_{\infty}^{CS}(t) + \Delta\varphi_{\infty}^{CC} + \Delta\varphi_{\infty}^{Cg}$$

$$\Phi_{\infty}^{CS}(t) = \frac{1}{4\pi R^2 \alpha} \sum_{S=1}^{N} \left\{ \frac{I_{CS0}}{\lambda_{S1}} \left(1 - \int_{0}^{1} e^{-\frac{t}{\tau_{S1} u}} du \right) - \frac{I_{CS0}}{\lambda_{S1}} \left(1 - \int_{0}^{1} e^{-\frac{t}{\tau_{S0} u}} du \right) \right\}$$

$$\Delta \varphi_{\infty}^{CC}(t) = \frac{1}{R^{2} \alpha} \sum_{S=1}^{N} \left[\sum_{n=1}^{N_{SC}} \Delta Q_{SC} E_{1} \left(\frac{t - nT_{S}^{C}}{\tau_{S0}} \right) - \sum_{n=1}^{N_{SC}} \Delta Q_{SC} E_{1} \left(\frac{t - nT_{S}^{C}}{\tau_{S1}} \right) \right]$$

$$\Delta \varphi_{\infty}^{C_g}(t) = \frac{1}{R^2 \alpha} \sum_{S=1}^{N} \left[\sum_{n=1}^{N_{S_g}} \Delta Q_{Sg} E_1 \left(\frac{t - nT_S^g}{\tau_0} \right) - E_1 \left(\frac{t - nT_S^g}{\tau_{Si}} \right) \right]$$

$$E_{1}(x) = \int_{1}^{\infty} e^{-xu} \frac{du}{u}, \qquad \lambda_{S1} = \lambda_{0} e^{\alpha(r_{S1} - R)}, \qquad \lambda_{S0} = \lambda_{0} e^{\alpha(r_{S0} - R)}$$

$$\tau_{S1} = (4\pi\lambda_{S1})^{-1}, \qquad \tau_{S0} = (4\pi\lambda_{S0})^{-1}$$
(5)

where τ_{S1} and τ_{S0} are times of electric relaxation in the regions of location of main electric charges of the thunderstorm generator and $\tau_0 = (4\pi\lambda_0)^{-1}$.

The first term in (5) for $\varphi_{\infty}(t)$ characterized the ionospheric potential part that is formed when the electrization current I_{CS} of thunderstorm current generators is switched on. The second term is

responsible for the ionospheric potential variations related to cloud-to-gloud discharges. Finally, the third term $\Delta \phi_{\infty}^{Cg}(t)$ characterizes potential variations during a cloud-to-ground discharge.

Intracloud discharges decrease the ionospheric potential. This was for the first time referred to by Hill [1971]. At the same time, negative cloud-to-ground discharges increase the potential of the ionosphere, and positive such discharges decrease this potential. At $t \gg \tau_{S1}$ and $t \gg \tau_{S0}$, the ionospheric potential $\phi_{\infty}(t)$ related to the charge current of thunderstorm generators I_{CS0} tends to the stationary value ϕ_{∞}^{0} defined by the expression

$$\varphi_{\infty}^{0} = \frac{1}{4\pi R^{2} \alpha} \sum_{S=1}^{N} I_{CS0} \left(\frac{\lambda}{\lambda_{S1}} - \frac{\lambda}{\lambda_{S0}} \right) = \frac{1}{4\pi \alpha} \sum_{S=1}^{N} (|Q_{S1}| - Q_{S0})$$

$$Q_{S0} = \frac{I_{CS0}}{4\pi \lambda(r_{S0})}, \qquad |Q_{S1}| = \frac{I_{CS0}}{4\pi \lambda(r_{S1})}.$$
(6)

We now obtain the φ_{∞}^0 values using the expression (6). As was mentioned in a number of works [Muhleisen, 1977; Roble and Tzur, 1986] using experimental data, to maintain the ionospheric potential $\varphi_{\infty}^0 = (250\text{-}300)$ kV, 2000 thunderstorms should simultaneously proceed over the entire globe. In a cloud dipole model, usually $Q_{S1} < 0$ and $Q_{S0} > 0$. Since $\lambda(r_{S0}) > \lambda(r_{S1})$, from this it follows that $|Q_{S1}| > Q_{S0}$ and $\varphi_{\infty}^0 > 0$.

At $\alpha = (0,2\text{-}0,3) \text{ km}^{-1}$, $R = 6,4\text{x}10^6 \text{ m}$, and N = 2000, from (6) we obtain that $\phi_{\infty}^0 = (220\text{-}140) \text{ kV}$ for $|Q_{SI}| - Q_{S0} = 100 \text{ C}$ for all thunderstorm generators and $\phi_{\infty}^0 = (330\text{-}220) \text{ kV}$ for $|Q_{SI}| - Q_{S0} = 150 \text{ C}$. In this case, if $\phi_{\infty}^0 = 330 \text{ kV}$, then $Q_{S0} = 30 \text{ C}$ and $Q_{S1} = -180 \text{ C}$. The selected differences between thundercloud electric charges before a lightning do not contradict the experimental data [Rutledge et al., 1990] and model calculations [Latham and Dye, 1989].

We now estimate the variations in the ionospheric potential φ_{∞}^{CS} related to cloud-to-ground discharges on the assumption that the discharge is directed to the ground from the lower negatively charged edge of a cloud. Assume that all N thunderclouds exert discharge of this type at a certain instant t ($t >> \tau_{S0}$, $t >> \tau_{S1}$). In such a case, we obtain the following expression for the potential at instant $t = T_0$ and at $\Delta Q_{Sg} = \Delta Q$:

$$\Delta \varphi_{\infty}^{C_g} = \sum_{S=1}^{N} \Delta Q \frac{z_{S1}}{R^2}, \qquad z_{S1} = r_{S1} - R,$$
 (7)

and at $t - T_0 = \tau_0$:

$$\Delta \varphi_{\infty}^{C_g} = \sum_{S=1}^{N} \Delta Q \frac{z_{S1}}{R^2} \left[E_1(1) - E_1\left(\frac{\tau_0}{\tau_{S1}}\right) \right]. \tag{8}$$

For $\Delta Q = \Delta Q_{Sg} = 10$ C, $\alpha = (0,2-0,3)$ km⁻¹, T = 2000, $\tau_0 = 600$ s, $\tau_{S1} = 100$ s and $z_{S1} = 6$ km, we obtain that $\Delta \varphi_{\infty}^{Cg}(\tau_0) = 26,4$ kV at $t = T_0$, then decreases, and becomes equal to $\Delta \varphi_{\infty}^{Cg}(3,2-4,8)$ kV at $t - T_0 = \tau_0$.

According to Morozov [2005], the electric field strength near the Earth's surface at $\ln \frac{\tau_0}{\tau} >> 1$ is

$$E_{z}(z=0,t) = -\sum_{S=1}^{N} \frac{1}{R^{2}} \Delta Q \frac{z_{S1}}{h(t)}; \qquad h(t) = \frac{1}{\alpha} \ln \frac{\tau_{0}}{t}.$$
 (9)

For $t \gg \tau_{S1}$ ($\tau_{S1} = 100$ s) and $t \geq \tau_0$, we have the asymptotic expression for E_z :

$$E_{z}(z=0,t) = -\frac{1}{R^{2}} \sum_{S=1}^{N} \Delta Q e^{-\frac{1}{\tau_{0}}} \left\{ 1 - \frac{\tau_{S1}}{\tau_{0}} \int_{0}^{\infty} \frac{(u+1)e^{-\frac{1}{\tau_{0}}u}}{u(\ln^{2}u + \pi^{2})} \right\}.$$
 (10)

The numerical estimations indicate that for $\frac{\tau_{S1}}{\tau_0}$ < 1 the expression in brackets is approximately equal to unity.

From Eq.(9) we obtain that $E_z = 0.9 \text{ V m}^{-1}$ for t = 0.1 s, $\alpha = 0.3 \text{ km}^{-1}$, and h(t) = 29 km and $E_z = 1.24 \text{ V m}^{-1}$ for t = 1 s and h(t) = 21.3 km.

At the same time, from (10) we obtained that $E_z = 0.6 \text{ V m}^{-1}$ for $t = 2 \tau_0$.

3. CONCLUSIONS

Variations of the ionospheric potential due to intracloud lightnings and positive cloud-to-ground lightnings are negative. Negative cloud-to-ground lightnings produce the positive variations of ionospheric potential. This variations is small ($\leq 10\%$). The input of negative cloud-to-ground lightnings in the variations of the electrical field strength is also small ($\leq 2\%$).

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