On the Description of Thunderstorm Generators and Its Relation to the Impact of Large-Scale Conductivity Inhomogeneities on the Ionospheric Potential

Evgeny A. Mareev[1*], Nikolay N. Slyunyaev[1], Alexey V. Kalinin[1,2] and Artem A. Zhidkov[1,2]

[1]Institute of Applied Physics RAS, Nizhny Novgorod, Russia
[2]Lobachevsky State University of Nizhny Novgorod, Nizhny Novgorod, Russia

ABSTRACT: Two alternative approaches to the description of thunderstorms in models of the global electric circuit (GEC) are considered, one treating thunderstorms as current sources, and the other, as voltage sources; it is shown that the two approaches are equally convenient in simple equivalent circuit models of the GEC, but in more realistic continuous three-dimensional spherical models the current-source approach proves more natural and useful. Within the current-source approach a number of simple model problems are analysed so as to illustrate the effect of conductivity inhomogeneities on the ionospheric potential; it is shown that taking account of the conductivity reduction inside thunderclouds leads to a substantial increase in the ionospheric potential, while the increase of conductivity above thunderstorms does not lead to a significant change in the ionospheric potential. The two approaches to the description of generators of the GEC are compared from the perspective of the ionospheric potential variation due to the increase of conductivity in the upper atmosphere; it is shown that the two parameterisations of thunderstorms yield qualitatively different results.

INTRODUCTION

Much attention has been given over the years to different approaches to modelling the global electric circuit (GEC) and particularly to the influence of conductivity inhomogeneities on its characteristics [*Zhou and Tinsley*, 2010; *Rycroft and Harrison*, 2012; *Williams and Mareev*, 2014]. Notwithstanding that most existing models of the GEC are based on the same equations of electrodynamics, the problem of parameterising generators of the GEC is still important and open to discussion. Historically, two different approaches to this issue have been widely used. Within one of these approaches thunderstorms are regarded as current sources [e.g., *Volland*, 1984], and within the other, as voltage sources [e.g., *Markson*, 1978]. Although there are certain reasons in favour of both methods of description [*Willett*, 1979], in most cases the current-source approach seems to be a more natural framework for representing thunderclouds.

In this paper we compare the two approaches to the description of thunderstorms and discuss some model problems concerning the influence of large-scale conductivity inhomogeneities on the ionospheric potential.

TWO APPROACHES TO THE DESCRIPTION OF THUNDERSTORM GENERATORS

In order to compare different approaches to the description of thunderstorm generators and to study the impact of conductivity inhomogeneities on the ionospheric potential, we will use two models of the GEC. One of them is a spherical model based on Maxwell's equations in a continuous medium, and the other is a simple multi-column model in which the GEC is replaced by the equivalent circuit consisting of resistors and current or voltage sources.

*Corresponding author, email: evgeny.mareev@gmail.com, Postal address: Institute of Applied Physics RAS, 46 Ulyanova St., 603950 Nizhny Novgorod, Russia.

In early studies of atmospheric electricity generators of the GEC were often described as point current sources [*Kasemir*, 1959; *Hays and Roble*, 1979; *Willett*, 1979]. Although such an approximation often yields realistic conclusions, the real thunderstorms are distributed rather than localised, and therefore continuous description of generators seems more appropriate. The current-source description of thunderstorms can be easily made continuous by employing the notion of the external current density $\mathbf{J}^{\mathrm{ext}}$, which enters into Ohm's law

$$\mathbf{J} = \sigma\mathbf{E} + \mathbf{J}^{\mathrm{ext}}, \tag{1}$$

where $\mathbf{J}$ is the current density, $\mathbf{E}$ is the electric field and σ stands for the conductivity.

The spherical model of the GEC which we use here was developed by *Kalinin et al.* [2014]. The atmosphere is represented by a shell confined between the Earth's surface $r = r_{\min}$ and the lower boundary of the ionosphere $r = r_{\max}$ (here $(r,\, \theta,\, \psi)$ are spherical coordinates whose origin coincides with the centre of the Earth). The equations governing the spatial distribution of the electric potential $\phi(r,\, \theta,\, \psi)$ read as follows:

$$\operatorname{div}(\sigma \operatorname{grad}\phi) = \operatorname{div}\mathbf{J}^{\mathrm{ext}}, \tag{2}$$

$$\oint_{r=r_{\min}} (\sigma \operatorname{grad}\phi)\, d\mathbf{S} = \oint_{r=r_{\min}} \mathbf{J}^{\mathrm{ext}}\, d\mathbf{S}, \tag{3}$$

$$\phi|_{r=r_{\min}} = 0, \qquad \phi|_{r=r_{\max}} = V_{\mathrm{i}}, \tag{4}$$

where V_{i} stands for the ionospheric potential. These equations precisely correspond to the general Maxwell's equations for the electric and magnetic fields in the atmosphere together with the relation (1). One of the most important aspects of these equations is that the ionospheric potential is not explicitly specified but is uniquely determined from the solution $\phi(r,\, \theta,\, \psi)$. It was shown that the problem (2)–(4) is well-posed, and once $\sigma(r,\, \theta,\, \psi)$ and $\mathbf{J}^{\mathrm{ext}}(r,\, \theta,\, \psi)$ are known, the potential distribution can be calculated numerically by means of the Galerkin method.

Simple multi-column models of the GEC based on the concept of the equivalent circuit are another convenient tool to compare different approaches to the description of thunderstorms. In such models the entire atmosphere is divided into two or more columns, some corresponding to thunderstorm regions, where the current flows upwards, and others corresponding to fair weather regions, where the current flows downwards. Then different regions are replaced by equivalent resistors and current or voltage sources, and thus the real atmospheric electric system is replaced by an equivalent circuit [*Markson*, 1978; *Makino and Ogawa*, 1984; *Odzimek et al.*, 2010]. Once the characteristics of these resistors and current or voltage sources are known, all the currents and voltages in the circuit (including the ionospheric potential) can be calculated using Kirchhoff's laws. The simplest example of such a model is shown in Fig. 1; there are only two columns in it, one of which contains all thunderstorms and the other contains all the fair-weather regions.

It can be shown that multi-column models of the GEC and their corresponding equivalent circuit models can be regarded as special cases of the general spherical model described above. More precisely, if the atmosphere is divided into several columns such that the conductivity and the external current density in each of these columns are functions of r alone and besides only the radial component of the external current density is non-zero, then the corresponding multi-column model turns out to be an approximation of the spherical model in which the currents flowing through the side surfaces of these columns are neglected (in other words, what is neglected are the derivatives with respect to θ and ψ in the equation (2) within each column).

Although equivalent circuit models are simple and convenient, they are not very useful for quantitative estimates. The comparison of the 'exact' potential distribution found by numerical solution of (2)–(4) and its approximation found from the corresponding equivalent circuit model shows that there is a substantial

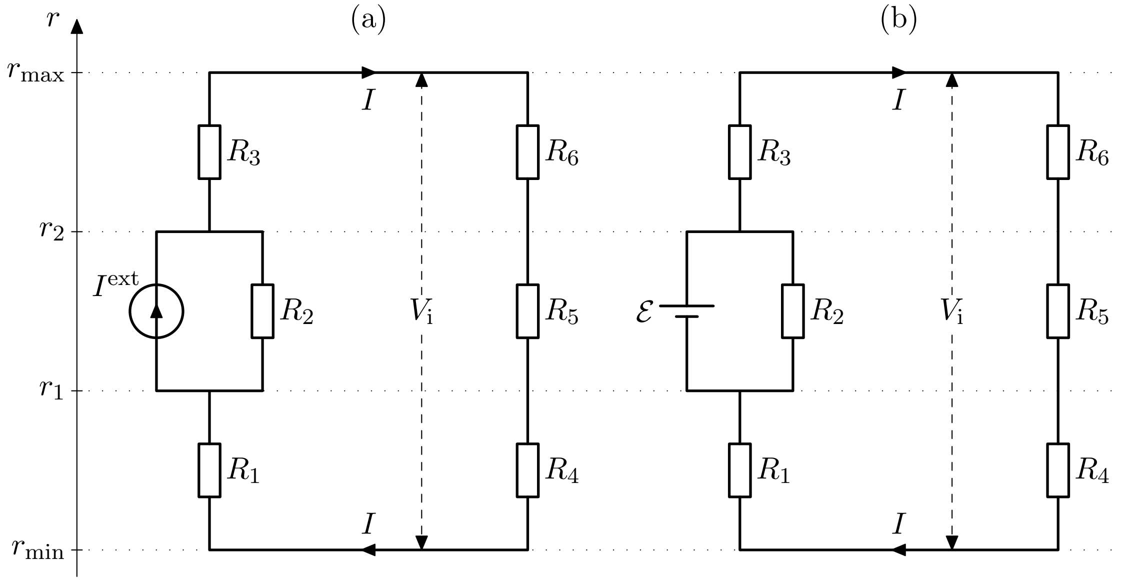


Figure 1: (a) The electric circuit equivalent to a two-column model of the GEC within the current-source approach. (b) The electric circuit equivalent to a two-column model of the GEC within the voltage-source approach.

difference between the two results, provided that the total area of the side surfaces of the columns into which the atmosphere is divided is large enough. More precisely, equivalent circuit models cannot distinguish between a number of separate identical thundercloud columns and all these columns combined together into a single column (as shown in Fig. 1). Therefore such models yield the same value of the ionospheric potential for a single large thundercloud and for a number of small ones with the same parameters and total area. However, numerical simulation shows that the current through the side surface of each column can be neglected only if the characteristic 'horizontal' scale of this column is much greater than the characteristic vertical scale of the atmosphere, $r_{\max} - r_{\min}$, and whereas this condition is fulfilled in the case of a single large thundercloud, it is not satisfied in the more realistic case of small thunderclouds. Therefore in most cases the equivalent circuit approach is unsuitable for quantitative analysis. Nevertheless, it yields many qualitatively correct results; for example, it explains the mechanisms by which various conductivity inhomogeneities affect the ionospheric potential.

In equivalent circuit models the current-source and voltage-source approaches are equally convenient. Since the external current inside a thundercloud and the voltage between its top and bottom are related, the main question is which quantity should be kept constant when we study the influence of conductivity variations on the GEC. Let us illustrate this situation by the example shown in Fig. 1. Denoting the current in the circuit by I and the voltage across the current or voltage source by $\mathcal{E}$, we get two expressions for the ionospheric potential:

$$V_{\text{i}} = \mathcal{E} - I\,(R_1 + R_3) = I\,(R_4 + R_5 + R_6), \tag{5}$$

whence, eliminating I, we arrive at the formula

$$V_{\text{i}} = \frac{\mathcal{E}}{R_1 + R_3} \bigg/ \left(\frac{1}{R_1 + R_3} + \frac{1}{R_4 + R_5 + R_6} \right). \tag{6}$$

This formula describes the ionospheric potential in terms of $\mathcal{E}$ and the resistances R_1, R_2, …, R_6 within the voltage-source approach. In order to obtain its counterpart within the current-source approach, we should express $\mathcal{E}$ in terms of I^{ext},

$$\mathcal{E} = \big(I^{\text{ext}} - I\big) R_2, \tag{7}$$

and then eliminate $\mathcal{E}$ and I from (5) and (7); the resulting formula is

$$V_{\rm i} = \frac{I^{\rm ext} R_2}{R_1 + R_2 + R_3} \Bigg/ \left(\frac{1}{R_1 + R_2 + R_3} + \frac{1}{R_4 + R_5 + R_6} \right). \tag{8}$$

Note that $I^{\rm ext}$ and $\mathcal{E}$ are related through (6) and (8). Given the external current $I^{\rm ext}$, we can always find the corresponding voltage $\mathcal{E}$, and vice versa. However, the two approaches differ as to which quantity is independent of the resistances, $I^{\rm ext}$ or $\mathcal{E}$. As the conductivity varies, the resistances change; on the contrary, $I^{\rm ext}$ within the current-source approach and $\mathcal{E}$ within the voltage-source approach remain constant. Hence, in order to study the influence of conductivity inhomogeneities on the ionospheric potential, one should use the formula (8) within the current-source approach and the formula (6) within the voltage-source approach. Obviously, a similar conclusion can be drawn for more general multi-column models than the one shown in Fig. 1.

Although the current-source and voltage-source descriptions of generators are very similar for equivalent circuit models, this is not the case for the general spherical model. Indeed, within the current-source approach it suffices to specify the external current density distribution $\mathbf{J}^{\rm ext}(r,\, \theta,\, \psi)$ in order to find the potential distribution (and the ionospheric potential), and the corresponding problem (2)–(4) is well-posed. On the other hand, there is no natural way to represent a real distributed thunderstorm in a continuous medium as a voltage source, except for thunderstorms of simple geometry and internal structure. Since three-dimensional spherical models of the GEC are more realistic than simple multi-column models, this is another critical advantage of the current-source description of generators.[†]

The next section is dedicated to the analysis of certain model problems regarding the effect of large-scale conductivity inhomogeneities in the atmosphere on the ionospheric potential. The current-source approach is used throughout this section, since it is more natural and convenient for our spherical model.

INFLUENCE OF LARGE-SCALE CONDUCTIVITY INHOMOGENEITIES ON THE IONOSPHERIC POTENTIAL WITHIN THE CURRENT-SOURCE APPROACH

Influence of the conductivity reduction inside thunderclouds

It is widely recognised that the conductivity inside thunderclouds is lower than that of the surrounding air [e.g., *Zhou and Tinsley*, 2010]. This hypothesis is reinforced by both direct measurements and theoretical considerations, the latter attributing the effect to the attachment of ions to hydrometeors.

A very important question is how the conductivity reduction inside thunderclouds affects the ionospheric potential $V_{\rm i}$. It is convenient to study the contributions to $V_{\rm i}$ from different thunderclouds rather than $V_{\rm i}$ itself. Strictly speaking, when we calculate the contribution $\delta V_{\rm i}$ to the ionospheric potential from a single thundercloud, we should account for all conductivity inhomogeneities caused by other thunderclouds, inasmuch as otherwise the superposition principle would not hold. However, as long as thunderclouds cover only a small portion of the Earth's surface and different thunderclouds are located rather far from each other, one can neglect those inhomogeneities and only allow for the conductivity reduction inside the thundercloud in question.

In order to estimate the effect of the conductivity reduction inside a single thundercloud on its contribution to the ionospheric potential, let us consider a model thundercloud occupying the region (in the spherical shell atmosphere)

$$\{(r,\, \theta,\, \psi)\colon r_1 \le r \le r_2,\, \theta \le \xi\}, \tag{9}$$

where $r_{\min} < r_1 < r_2 < r_{\max}$ (see Fig. 2).

[†]Strictly speaking, in order to allow for different mechanisms of the charge separation in thunderclouds, we must take into consideration that $\mathbf{J}^{\rm ext}$ may depend on $\mathbf{E}$; however, this matter is beyond the scope of this paper.

Let us first employ the more accurate spherical model. We suppose that the external current density distribution corresponding to the thundercloud (9) is of the form

$$J_r^{\text{ext}}(r,\,\theta,\,\psi) = \begin{cases} J_0, & r_1 \le r \le r_2,\ \theta \le \xi, \\ 0, & \text{otherwise}, \end{cases}$$

$$J_{\theta,\psi}^{\text{ext}}(r,\,\theta,\,\psi) = 0 \quad \text{for all } (r,\,\theta,\,\psi),$$

and we assume that the conductivity distribution is of the form

$$\sigma(r,\,\theta,\,\psi) = \begin{cases} X \cdot \sigma^0(r), & r_1 \le r \le r_2,\ \theta \le \xi, \\ \sigma^0(r), & \text{otherwise}, \end{cases}$$

where

$$\sigma^0(r) = \sigma_0 \exp\left(\frac{r - r_{\min}}{H}\right), \tag{10}$$

H is a scale height and the parameter $X \le 1$ serves as the measure of the conductivity reduction inside the thundercloud.

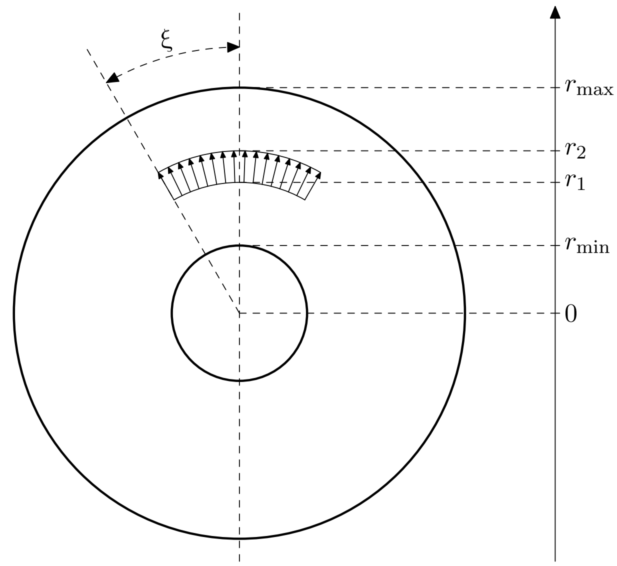


Figure 2: The geometry of a model thundercloud.

In Fig. 3 the solid line shows the dependence of the ionospherical potential on the parameter X, as obtained from the numerical solution of the equations (2)–(4) with $\sigma(r,\,\theta,\,\psi)$ and $\mathbf{J}^{\text{ext}}(r,\,\theta,\,\psi)$ described above by means of the Galerkin method. We observe that the more the conductivity inside thundercloud is reduced, the larger is its contribution to the ionospheric potential δV_{i}. This trend can be explained using the corresponding simple two-column model.

Let us consider the equivalent circuit model shown in Fig. 1a. To establish the correspondence between this model and the two-column approximation of the spherical model considered above, we choose the following parameters‡:

$$R_1 = \frac{1}{\gamma S} \int_{r_{\min}}^{r_1} \frac{dr}{\sigma^0(r)}, \tag{11}$$

$$R_2 = \frac{1}{\gamma S X} \int_{r_1}^{r_2} \frac{dr}{\sigma^0(r)}, \tag{12}$$

$$R_3 = \frac{1}{\gamma S} \int_{r_2}^{r_{\max}} \frac{dr}{\sigma^0(r)}, \tag{13}$$

$$R_4 = \frac{1}{(1-\gamma)\,S} \int_{r_{\min}}^{r_1} \frac{dr}{\sigma^0(r)}, \tag{14}$$

$$R_5 = \frac{1}{(1-\gamma)\,S} \int_{r_1}^{r_2} \frac{dr}{\sigma^0(r)}, \tag{15}$$

$$R_6 = \frac{1}{(1-\gamma)\,S} \int_{r_2}^{r_{\max}} \frac{dr}{\sigma^0(r)}, \tag{16}$$

$$I^{\text{ext}} = \gamma S J_0, \tag{17}$$

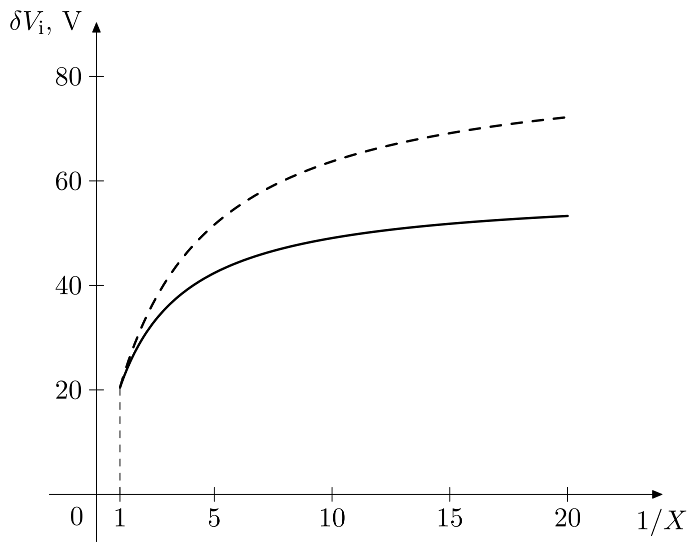


Figure 3: The contribution from a single thundercloud to the ionospheric potential, as according to numerical modelling (the solid line) and the formula (19) (the dashed line). The parameters are as follows: $r_{\min} = 6370\,\text{km}$, $r_{\max} = r_{\min} + 70\,\text{km}$, $r_1 = r_{\min} + 5\,\text{km}$, $r_2 = r_{\min} + 10\,\text{km}$, $\xi = \pi/4000$ (which corresponds to the thundercloud diameter of approximately $10\,\text{km}$), $J_0 = 3 \cdot 10^{-9}$ A m^{-2}, $\sigma_0^{-1} = 3 \cdot 10^{13}\,\Omega$ m and $H = 6\,\text{km}$.

‡We use the relation $r_{\max} - r_{\min} \ll r_{\min}, r_{\max}$.

where S stands for the Earth's surface and $\gamma = (1 - \cos\xi)/2$ denotes the portion of the Earth's surface covered by the thundercloud (9). Since we use the current-source approach, the ionospheric potential is described by the formula (8), which can be written in the form

$$\delta V_{\mathrm{i}} = \frac{I^{\mathrm{ext}}}{\dfrac{1}{R_4 + R_5 + R_6} + \left(1 + \dfrac{R_1 + R_3}{R_4 + R_5 + R_6}\right)\dfrac{1}{R_2}}. \tag{18}$$

Since $R_2 \propto 1/X$, this means that the ionospheric potential is a fractional linear function of X. Substituting the expressions for the total external current I^{ext} and the resistances $R_1, R_2, \ldots, R_6$ in terms of $\sigma^0(r)$, J_0 and X into (18), we arrive at the formula

$$\delta V_{\mathrm{i}} = \frac{\gamma \delta V_0}{(1-\gamma) K + (1 - (1-\gamma) K) X}, \tag{19}$$

where

$$\delta V_0 = J_0 \int_{r_1}^{r_2} \frac{dr}{\sigma^0(r)}, \qquad K = \int_{r_1}^{r_2} \frac{dr}{\sigma^0(r)} \Big/ \int_{r_{\min}}^{r_{\max}} \frac{dr}{\sigma^0(r)}.$$

The ionospheric potential given by (19) is shown by the dashed line in Fig. 3.

By comparing the two curves in Fig. 3, we observe that their shapes are similar; furthermore, the two functions yield the same δV_{i} at $X = 1$, which is because in this case the formula (19) is exact, as was shown by *Kalinin et al.* [2014]. However, the more is the conductivity reduction, the greater is its relative error. This is another illustration of the fact that although the equivalent circuit models yield qualitatively correct results, they are not useful for quantitative analysis.

It is easy to conclude from Fig. 3 that the ionospheric potential substantially depends on the conductivity reduction in thunderclouds. Therefore it is extremely important to allow for this phenomenon in quantitative studies of atmospheric electricity. For example, if we know the contribution δV_{i} to the ionospheric potential from a typical thundercloud, we can estimate the total number of thunderclouds by dividing the total ionospheric potential V_{i} by δV_{i}; assuming that the parameters of a typical thundercloud are the same as listed in Fig. 3 caption and the total ionospheric potential is equal to $240\,\mathrm{kV}$, we obtain that the number of thunderstorms is about 11700 if the conductivity reduction is not taken into account and about 4900 if a tenfold reduction is assumed. However, this number substantially depends on the values of the external current density inside thunderclouds J_0 and the air conductivity at the Earth's surface σ_0, which in the example above were taken equal to $3 \cdot 10^{-9}\,\mathrm{A\,m^{-2}}$ and $1/3 \cdot 10^{-13}\,\Omega^{-1}\,\mathrm{m}^{-1}$ respectively. Actually the ratio J_0/σ_0 may be greater, and in consequence the number of thunderclouds in the atmosphere may be substantially smaller.

Influence of conductivity inhomogeneities outside thunderclouds

Certain trends in the variation of the ionospheric potential are often attributed to various natural phenomena (e.g., ionising radiation due to solar flares) and anthropogenic factors (e.g., nuclear radiation caused by weapons testing and accidents in power plants), and conductivity variations are usually regarded as one of the possible mechanisms for the effect that these factors have on the GEC [e.g., *Markson*, 2007]. Both numerical solution of the spherical model equations and qualitative estimation by means of equivalent circuit models show that in most cases conductivity inhomogeneities inside thunderclouds lead to a much more significant change in the ionospheric potential than conductivity inhomogeneities of the same order outside thunderclouds.

In order to illustrate this idea, let us consider a model problem where all thunderclouds are situated near the equator and the conductivity is reduced inside them and increased in the axisymmetric region situated above the cloud level in the Northern Hemisphere (see Fig. 4). More precisely, we suppose that thunderclouds occupy the entire region

$$\{(r,\, \theta,\, \psi)\colon r_1 \le r \le r_2,\, \xi \le \theta \le \pi - \xi\},$$

so the external current density distrbution is of the form

$$J_r^{\text{ext}}(r,\, \theta,\, \psi) = \begin{cases} J_0, & r_1 \le r \le r_2,\, \xi \le \theta \le \pi - \xi, \\ 0, & \text{otherwise}, \end{cases}$$

$$J_{\theta,\,\psi}^{\text{ext}}(r,\, \theta,\, \psi) = 0 \quad \text{for all } (r,\, \theta,\, \psi),$$

and we assume that the conductivity distribution is of the form

$$\sigma(r,\, \theta,\, \psi) = \begin{cases} X \cdot \sigma^0(r), & r_1 \le r \le r_2,\, \xi \le \theta \le \pi - \xi, \\ Y \cdot \sigma^0(r), & r_3 \le r \le r_4,\, \theta \le \chi, \\ \sigma^0(r), & \text{otherwise}, \end{cases}$$

where $\sigma^0(r)$ is described by (10) and $r_{\min} < r_1 < r_2 < r_3 < r_4 < r_{\max}$. Here the parameter X again means the degree of the conductivity reduction inside thunderclouds and the parameter Y measures the conductivity increase in the region

$$\{(r,\, \theta,\, \psi)\colon r_3 \le r \le r_4,\, \theta \le \chi\} \qquad (20)$$

(the hatched region in Fig. 4). Our choice of such a distribution of thunderclouds is motivated by the fact that in the real atmosphere most thunderstorms are located near the equator, and conductivity perturbations in the region (20) are supposed to represent typical conductivity inhomogeneities caused by solar flares and precipitation of energetic particles.

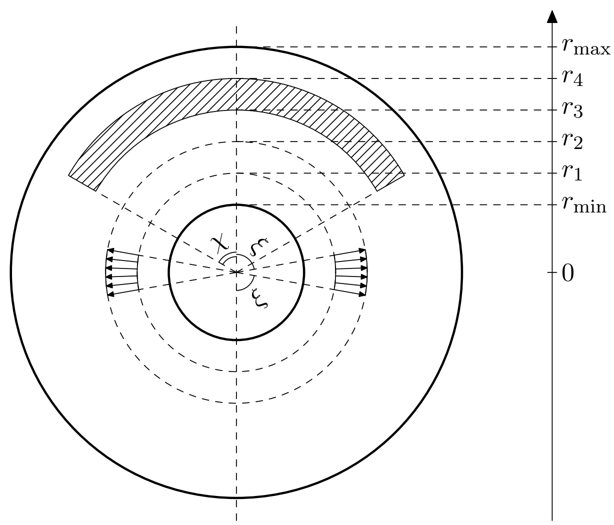
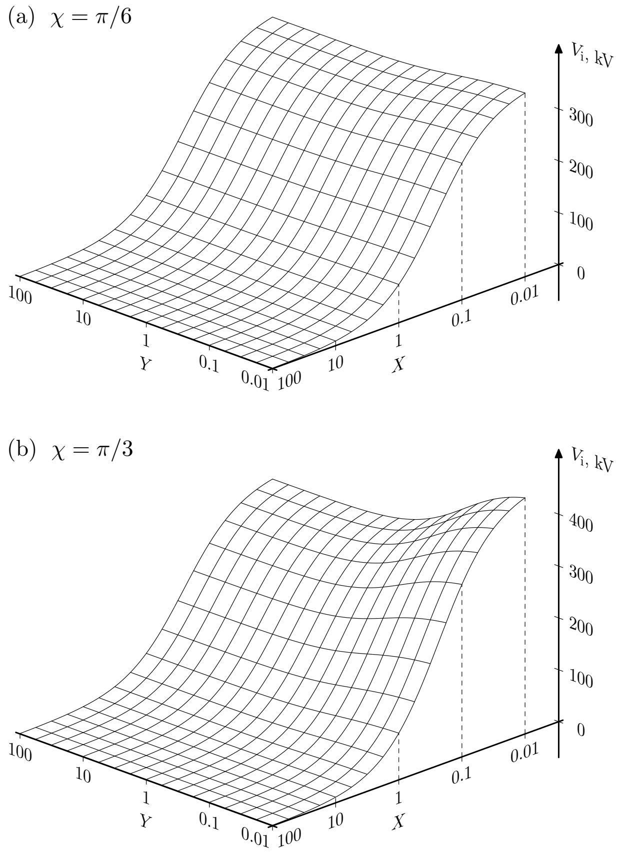


Figure 4: Geometry of the model problem.

It is crucial to note that here we have replaced the real distribution of thunderclouds with a single continuous region $\{(r,\, \theta,\, \psi)\colon r_1 \le r \le r_2,\, \xi \le \theta \le \pi - \xi\}$. As it was stated in the previous section, even if the total external current and the total area overcast by thunderclouds are chosen to match the real values, there still may be a considerable difference between the potential distributions produced by a number of small separate thunderclouds and a ‘single thundercloud’ region as described above, since in the latter case all the ‘fringe’ effects concerning the currents which return to the Earth’s surface through the middle atmosphere without reaching the ionosphere are neglected, whereas these effects must be taken into account in order to detemine the potential distribution quantitatively. However, such an approximation is still useful for qualitative analysis, while the axisymmetric geometry substantially simplifies and speeds up numerical computations.

In Fig. 5 the dependence of the ionospheric potential on the parameters X and Y, obtained from the numerical solution of (2)–(4), is shown for three different values of χ. The assumption that the conductivity is reduced inside thunderclouds and increased inside the region (20) is equivalent to the conditions $X \le 1$, $Y \ge 1$. We observe that for such X and Y the ionospheric potential V_{i} increases substantially with



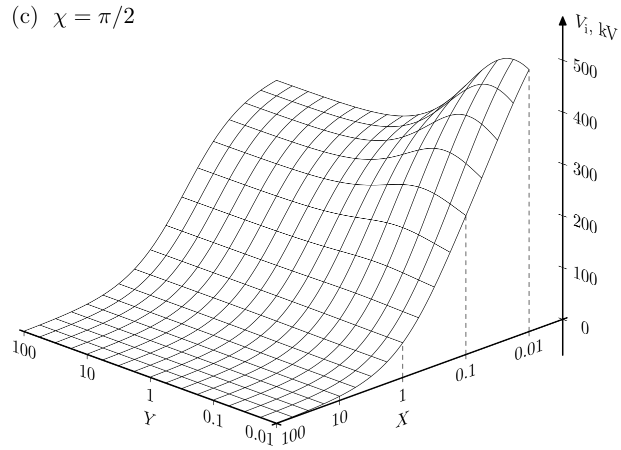


Figure 5: The ionospheric potential, obtained from numerical calculations, for different values of χ. The parameters are as follows: $r_{\min} = 6370\,\mathrm{km}$, $r_{\max} = r_{\min} + 70\,\mathrm{km}$, $r_1 = r_{\min} + 5\,\mathrm{km}$, $r_2 = r_{\min} + 10\,\mathrm{km}$, $r_3 = r_{\min} + 12\,\mathrm{km}$, $r_4 = r_{\min} + 25\,\mathrm{km}$, $\xi = 4\pi/9$ (which corresponds to the thundercloud region between 10°N and 10°S), $J_0 = 9 \cdot 10^{-12}\,\mathrm{A\,m^{-2}}$, $\sigma_0^{-1} = 3 \cdot 10^{13}\,\Omega\,\mathrm{m}$ and $H = 6\,\mathrm{km}$.

increasing X, but does not change much with increasing Y, whatever the value of χ may be. Note that in case $X \gg 1$ the ionospheric potential approaches zero, which is not surprising, since in this case almost all the current I^{ext} in the network of Fig. 1a does not reach ionosphere and flows back through R_2. In case $X \ll 1$ and $Y \ll 1$ the dependence $V_{\mathrm{i}}(X, Y)$ turns out to be very complicated (these values of X and Y are beyond the range of Fig. 5); however, this case does not correspond to any real physical problem.

Since in the model problem discussed above thunderclouds are combined into a single large region, the equivalent circuit approximation describes this problem with high precision. Using this approximation, one can derive an explicit formula expressing the ionospheric potential V_{i} in terms of X and Y, and it turns out that its relative error is very small for all values of X and Y. However, this formula is rather cumbersome and not very useful.

COMPARISON OF THE TWO APPROACHES TO THE DESCRIPTION OF THUNDERSTORM GENERATORS

As it was already mentioned, one of the most critical distinctions between the current-source and voltage-source approaches is that the former can be easily integrated into continuous three-dimensional spherical models, whereas there is no natural way to do so for the latter. On the other hand, this distinction does not hold for the corresponding equivalent circuit approximations, both of which are equally natural and useful. Therefore, since the equivalent circuit approach in most cases yields qualitatively correct results, it is convenient to employ this approach in order to clarify the difference between the two methods of the description of generators.

Let us consider a model problem corresponding to the circuit shown in Fig. 1. We suppose first that the conductivity is described by the function $\sigma^0(r)$, as defined in (10), except that inside thunderstorms this function is multiplied by the factor X. Within the current-source approach (Fig. 1a) we also suppose that the external current density (which is assumed to be purely radial) is equal to J_0 inside thunderstorms and equal to zero outside them. It is easy to see that in this case the resistances R_1, R_2, …, R_6 and the total external current $I^{\rm ext}$ in thunderstorms are described by (11)–(17), γ being the portion of the Earth's surface overcast by thunderclouds, and hence the ionospheric potential can be calculated using the formula (19) (in which $\delta V_{\rm i}$ should be replaced with $V_{\rm i}$). Taking $r_{\min} = 6370\,{\rm km}$, $r_{\max} = r_{\min} + 70\,{\rm km}$, $r_1 = r_{\min} + 5\,{\rm km}$, $r_2 = r_{\min} + 10\,{\rm km}$, $J_0 = 3 \cdot 10^{-9}$ A m^{-2}, $\sigma_0^{-1} = 3 \cdot 10^{13}\,\Omega$ m, $H = 6\,{\rm km}$ and $X = 0.1$, and requiring that $V_{\rm i} = 240\,{\rm kV}$, we find that $\gamma = 0.17\%$. By comparing (6) and (8) we obtain that the potential difference $\mathcal{E}$ across thunderstorms is equal to $1.04 \cdot 10^8$ V.

Suppose now that the conductivity doubles over the upper boundary of thunderclouds $r = r_2$. This means that R_3 and R_6 halve and all other resistances remain unchanged. Then within the current-source approach we keep $I_{\rm ext}$ constant and by (8) find that $\Delta V_{\rm i}/V_{\rm i} = -6.7\%$, and within the voltage-source approach we keep $\mathcal{E}$ constant and by (6) find that $\Delta V_{\rm i}/V_{\rm i} = 3.5\%$ (here $\Delta V_{\rm i}$ denotes the variation of $V_{\rm i}$ due to the doubling of conductivity). In other words, the ionospheric potential increases if thunderstorms are treated as voltage sources and decreases if they are regarded as current sources. Such a critical discrepancy reinforces the idea that the choice of the description of generators of the GEC is extremely important.

Using the voltage-source approach and a two-column model of the GEC, *Markson* [1978] estimated that a doubling of conductivity in the upper atmosphere leads to a 40% increase in the ionospheric potential. Furthermore, a more accurate estimation shows that actually the ionospheric potential in the situation analysed by Markson increases by 70%. The substantial difference between these findings and those of the preceding paragraph is explained by the fact that Markson assumed a 20-fold increase in conductivity below thunderclouds, and thus his analysis implies that R_3 is about one order of magnitude greater than R_1 (in terms of Fig. 1b), whereas without this assumption R_3 is several times less than R_1, and the ratio R_1/R_3 turns out to be critical for the estimation of the ionospheric potential within the voltage-source approach.[§]

Note that if X were equal to 1—that is to say, if the conductivity were not reduced inside thunderclouds,—then within the current-source approach the ionospheric potential would not change due to a doubling of conductivity in the upper atmosphere. Indeed, the formula (8) can be written as

$$V_{\rm i} = \frac{I^{\rm ext} R_2}{1 + \dfrac{R_1 + R_2 + R_3}{R_4 + R_5 + R_6}},$$

and from $X = 1$ it follows that $R_1/R_4 = R_2/R_5 = R_3/R_6$; therefore $V_{\rm i}$ does not change if we replace R_3 with $R_3/2$ and R_6 with $R_6/2$. This is an illustration of the general fact that a substantial increase in the ionospheric potential due to large-scale conductivity inhomogeneities in the atmosphere is impossible, if conductivity inhomogeneities in thunderstorm regions and fair-weather regions are distributed 'similarly'.

CONCLUSIONS

Although both the current-source and voltage-source approaches to the description of thunderstorms are convenient for simple equivalent circuit models, yet for more realistic continuous three-dimensional models, the former approach is more natural and useful. We have shown that the choice of the description of generators is a critical issue for modelling the GEC, since different approaches lead to substantially different

[§]Note, however, that within the current-source approach the ratio R_1/R_3 is not so important, if the conductivity reduction inside thunderstorms is taken into account, for in this case R_2 is much greater than both R_1 and R_3.

results. Within the current-source approach it is also important to allow for conductivity inhomogeneities inside thunderclouds, for they significantly affect the ionospheric potential. We have also shown that equivalent circuit models of the GEC (or, in other words, simple multi-column models) which have been widely used until now are rather simplistic and unsuitable for quantitative estimations. All these considerations must be taken into account in models of the GEC.

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