Analysis of Lightning Electromagnetic Fields at Near and Far Ranges

Yazhou Chen[1,2*], Hao Wang[2], Vladimir A. Rakov[1]

1. University of Florida, Gainesville, FL, USA
2. Shijiazhuang Mechanical Engineering College, Shijiazhuang, Hebei, China

ABSTRACT: The waveforms of lightning return stroke electromagnetic fields on ground are studied using the transmission-line model. Approximate expressions to calculate lightning electromagnetic fields at near and far ranges are presented. It is found that, the waveforms of lightning electric and magnetic fields in the time domain at near and far distances can be expressed approximately as the channel-base current waveform multiplied by a factor which is a function of return stroke speed *v*, the speed of light *c*, and the horizontal distance *r* between the return-stroke channel and the observation point on ground. The ranges at which the approximate expressions are valid are determined.

INTRODUCTION

Lightning is a common natural phenomenon, and it is estimated that some tens to 100 flashes, including cloud and ground flashes, occur every second on Earth. Cloud-to-ground flashes are of primary concern from the lightning protection point of view. The effects of lightning electromagnetic fields have caused significant problems in recent years [Rakov and Uman 2003; Rakov and Rachidi 2009; Miyazaki et al. 2010; Samaras et al. 2007; Thottappillil 2002]. It is important to investigate the characteristics of lightning electromagnetic fields using both theory and measurements in order to simulate the lightning electromagnetic fields in laboratory and to determinate the threat level to electric systems. Many works have been done in this regard [Hoole and Balasuriya 1993; Lin and Uman 1979; Jerauld et al. 2008; Barbosa et al. 2008; Lee et al. 2004; Hussein et al. 2008; Cooray et al. 2004; Uman et al. 2000; Jerauld et al. 2009; Rakov and Uman 1998; Cooray 1993]. Nucci et al. [1990] identified four characteristic features in the lightning electromagnetic fields at 1 to 200 km measured by Lin et al. [1979]. Rakov and Uman [1998] defined four classes of lightning return stroke models which can be used to relate electromagnetic fields to lightning current. Uman *et al*. [1975] demonstrated the approximate relationship between the lightning electromagnetic field at far ranges and channel-base current and showed that the far fields beyond a certain distance can be inferred from the channel-base current. Uman *et al*. [2002] found that the current derivative waveforms, the magnetic flux density derivative waveform, and electric field intensity derivative waveforms are essentially unipolar pulses that have similar wave shapes at 15 m from the triggered lightning channel. Similar structure was observed by Jerauld *et al*. [2007] in d*E*/d*t* at 15 m and 30 m and d*I*/d*t* waveforms produced by an unusual rocket-triggered lightning stroke, which involved a downward dart-stepped leader and a pronounced upward connecting leader. The above studies have

∗ Contact information:Yazhou Chen, University of Florida, Ganiesville, Florida, US, Email: chen_yazhou@sina.com

connected to some extent the waveforms of lightning electromagnetic field with that of the channel-base current. In this paper, in order to theoretically establish the relationship between the lightning electromagnetic field and the channel-base current in more detail, we will derive the approximate analytical expressions for the lightning electromagnetic fields on earth surface at near and far ranges, using the transmission line (TL) model and the dipole method. The ranges at which the approximate expressions are valid are established by comparing their predictions with the electromagnetic fields calculated using the exact expressions.

EXPRESSIONS FOR LIGHTNING ELECTROMAGNETIC FIELD AT GROUND LEVEL

The return-stroke channel can be regarded as a vertical antenna, as shown in Fig. 1. The transient current propagates along the vertical conductor (the lightning channel) above a perfectly conducting plane (the ground).

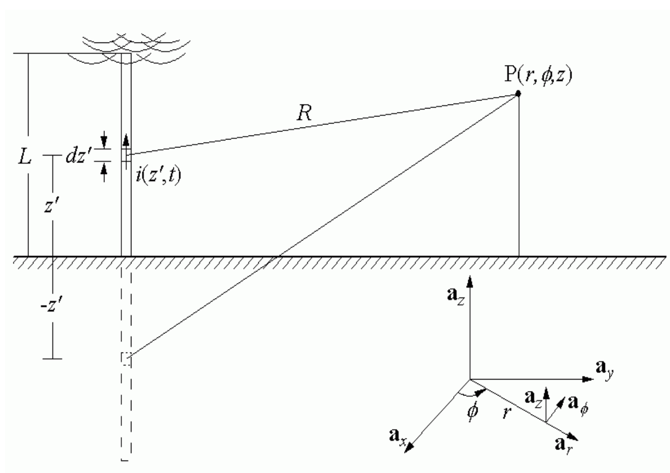

Fig. 1. The geometry used in calculating lightning return stroke electromagnetic fields.

According to the dipole method [Stratton 1941], the lightning electromagnetic fields due to an upward-moving return stroke at an observation point $\mathrm{P}(r,\phi,z)$ (in cylindrical coordinate system) can be described as follows:

$$E_r = \frac{1}{4\pi\varepsilon_0}\int_{-h_-}^{h_+}\left[\frac{3r(z-z')}{R^5}\int_{-\infty}^{t} i(z',\tau-R/c)\,\mathrm{d}\tau + \right.$$
$$\left.\frac{3r(z-z')}{cR^4}i(z',t-R/c)+\frac{r(z-z')}{c^2R^3}\frac{\partial i(z',t-R/c)}{\partial t}\right]\mathrm{d}z' \qquad (1)$$

$$E_z = \frac{1}{4\pi\varepsilon_0}\int_{-h_-}^{h_+}\left[\frac{2(z-z')^2-r^2}{R^5}\int_{-\infty}^{t} i(z',\tau-R/c)\,\mathrm{d}\tau + \right.$$
$$\left.\frac{2(z-z')^2-r^2}{cR^4}i(z',t-R/c)-\frac{r^2}{c^2R^3}\frac{\partial i(z',t-R/c)}{\partial t}\right]\mathrm{d}z' \qquad (2)$$

$$H_\phi = \frac{1}{4\pi}\int_{-h_-}^{h_+}\left[\frac{r}{R^3}i(z',t-R/c)+\frac{r}{cR^2}\frac{\partial i(z',t-R/c)}{\partial t}\right]\mathrm{d}z' \tag{3}$$

Where $R=\sqrt{z'^2+r^2\text{-}2zz'}$ is the distance between the observation point and the element dz' in the channel at height z', L is the length of the return-stroke channel, h_+ and h_- are the heights of the current front as "seen" by the observer at time t in the channel and in its image, respectively, c is the speed of light, i is the return-stroke current, E_r is the radial electric field, E_z is the vertical electric field, H_Φ is the azimuthal magnetic field. In equations (1) and (2), the first term represents the electrostatic field, the second term is the induction electric field, and the third term is the radiation electric field. In equation (3), the first term represents the induction component of the magnetic field and the second term is the radiation component of the magnetic field.

In this paper, we focus on the electromagnetic fields at ground level (i.e., the height of the observation point z=0). Thus, $h_+=h_-=h$, E_r=0.

According to the TL model [Uman and McLain 1969], the return stroke current is given by

$$i(z',t) = u(t-z'/v)\cdot i(0,t-z'/v) \tag{4}$$

where $u(\xi)$ is the Heaviside function equal to unity for ξ>0 and zero otherwise, and v is the lightning current propagation speed along the lightning channel.

Substituting $i(z',t)$ given by (4) into (2) and (3), we can obtain each component of the electric and magnetic fields, and the total electric and magnetic fields as follows [Rubinstein and Uman 1989]:

Electrostatic field:

$$E_z(\text{electrostatic}) = \frac{i(z',t)}{2\pi\varepsilon_0}\left\{\frac{-th+\frac{2h^2}{v}+\frac{r^2}{v}}{\left(h^2+r^2\right)^{3/2}}-\frac{1}{rv}-\frac{1}{2cr}\left[\tan^{-1}\left(\frac{h}{r}\right)-\frac{3hr}{h^2+r^2}\right]\right\} \tag{5}$$

Induction electric field:

$$E_z(\text{induction}) = -\frac{i(z',t)}{4\pi\varepsilon_0 rc}\left[\tan^{-1}\left(\frac{h}{r}\right)-\frac{3hr}{h^2+r^2}\right] \tag{6}$$

Radiation electric field:

$$E_z(\text{radiation}) = -\frac{i(z',t)}{2\pi\varepsilon_0 c^2\left(h^2+r^2\right)^{3/2}}\frac{r^2}{\left(\frac{1}{v}+\frac{h}{c(h^2+r^2)^{1/2}}\right)} \tag{7}$$

Induction magnetic field:

$$H_\phi(\text{induction}) = \frac{i(z',t)}{2\pi}\frac{h}{r\left(h^2+r^2\right)^{1/2}} \tag{8}$$

Radiation magnetic field:

$$H_\phi(\text{radiation}) = \frac{i(z',t)}{2\pi}\frac{r}{\frac{c}{v}\left(h^2+r^2\right)+h\left(h^2+r^2\right)^{1/2}} \tag{9}$$

Total electric field at ground level:

$$E_z(r,t) = \frac{i(z',t)}{2\pi\varepsilon_0}\left\{\frac{-th+\frac{2h^2}{v}+\frac{r^2}{v}}{\left(h^2+r^2\right)^{3/2}} - \frac{1}{rv} - \frac{r^2}{c^2\left(h^2+r^2\right)^{3/2}\left(\frac{1}{v}+\frac{h}{c\sqrt{h^2+r^2}}\right)}\right\} \tag{10}$$

Total magnetic field at ground level:

$$H_\phi(r,t) = \frac{i(z',t)}{2\pi}\left[\frac{h}{r\left(h^2+r^2\right)^{1/2}} + \frac{r}{\frac{c}{v}\left(h^2+r^2\right)+h\left(h^2+r^2\right)^{1/2}}\right] \tag{11}$$

Where $h = \beta\frac{ct-\sqrt{(\beta ct)^2+r^2\left(1-\beta^2\right)}}{1-\beta^2}$, and $\beta = v/c$.

APPROXIMATE EXPRESSIONS FOR LIGHTNING ELECTROMAGNETIC FIELDS AND THEIR DERIVATIVES AT NEAR RANGES

Approximate expressions for lightning electromagnetic fields at near ranges

If we let $\partial f(z',t)/\partial t = i(z',t)$, then according to the TL model:

$$\begin{aligned}\int_{-\infty}^{t} i(z',\tau)\mathrm{d}\tau &= \int_{-\infty}^{t} u(\tau - z'/v)\cdot i(0,\tau - z'/v)\mathrm{d}\tau = \int_{z'/v}^{t} i(0,\tau - z'/v)\mathrm{d}\tau \\ &= \int_0^{t-z'/v} i(0,s)\mathrm{d}s = f(0,t)\Big|_0^{t-z'/v} = f(0,t-z'/v) - f(0,0) \\ &= F(t-z'/v).\end{aligned} \tag{12}$$

Thus, the current integration in the electrostatic field component of equation (2) is:

$$\int_{-\infty}^{t} i(z',\tau - R/c)\mathrm{d}\tau = F(t - R/c - z'/v). \tag{13}$$

Using the Taylor series expansion, $F(t-R/c-z'/v)$ can be written as:

$$\begin{aligned}F(t-R/c-z'/v) &= F(t-r/c) - F'(t-r/c)\times\left[(t-r/c)-(t-R/c-z'/v)\right] + o\left(c^{-2}\right) \\ &\approx F(t-r/c) - i(0,t-r/c)\left[(R-r)/c + z'/v\right].\end{aligned} \tag{14}$$

Substituting (13) and (14) into the first term of equation (2), we can obtain the first-level approximation for the electrostatic field component as:

$$E'_z(\text{electrostatic}) = \frac{1}{2\pi\varepsilon_0}\int_0^h \left[\frac{2R^2-3r^2}{R^5}\int_{-\infty}^t i(z',\tau - R/c)\,\mathrm{d}\tau\right]\mathrm{d}z'$$
$$= \frac{1}{2\pi\varepsilon_0}\int_0^h \frac{2R^2-3r^2}{R^5} F(t - R/c - z'/v)\,\mathrm{d}z'$$
$$\approx \frac{F(t-r/c)}{2\pi\varepsilon_0}\left[\frac{-h}{R(h)^3}\right] - \frac{i(0,t-r/c)}{2\pi\varepsilon_0}$$
$$\times\left\{\frac{1}{c}\left(\frac{rh}{R(h)^3} - \frac{3h}{2R(h)^2} + \frac{\tan^{-1}(h/r)}{2r}\right) + \frac{1}{v}\left(\frac{r^2}{R(h)^3} - \frac{2}{R(h)} + \frac{1}{r}\right)\right\}. \tag{15}$$

The induction field component contains c^{-1}. Substituting the Taylor series expansion of $i(z',t-R/c)$ in the second term of equation (2), we can obtain the first-level approximation for the induction electric field component as:

$$E'_z(\text{induction}) \approx \frac{i(0,t-r/c)}{2\pi\varepsilon_0 c}\int_0^h \frac{2R^2-3r^2}{R^4}\,\mathrm{d}z' = \frac{i(0,t-r/c)}{2\pi\varepsilon_0 c}\left[\frac{\tan^{-1}(h/r)}{2r} - \frac{3h}{2R(h)^2}\right] \tag{16}$$

The radiation electric field component at near ranges is very small and can be ingored. Thus, adding equations (15) and (16), we obtain the first-level approximation expression for the total electric field at near ranges as:

$$E'_z \approx -\frac{F(t-r/c)}{2\pi\varepsilon_0}\frac{h}{R(h)^3} - \frac{i(0,t-r/c)}{2\pi\varepsilon_0}\left[\frac{rh}{cR(h)^3} + \frac{1}{v}\left[\frac{r^2}{R(h)^3} - \frac{2}{R(h)} + \frac{1}{r}\right]\right] \tag{17}$$

Similarly, we can obtain the first-level approximate expression for the magnetic field at near ranges as:

$$H'_\phi \approx H'_\phi(\text{induction}) = \frac{1}{2\pi} i(0,t-r/c)\int_0^h \frac{r}{R^3}\,\mathrm{d}z' = \frac{1}{2\pi r} i(0,t-r/c)\frac{h}{R(h)} \tag{18}$$

As $r \ll L$ at near ranges, $R(h)$ gradually approaches h when the current propagates upward along the channel. Neglecting the R^{-1} term and R^{-3} term compared with r^{-1} term in equation (17), we can obtain the second-level approximate expressions for the electromagnetic field at near ranges as:

$$E''_z \approx -\frac{1}{2\pi\varepsilon_0 v r} i(0,t-r/c) \tag{19}$$

$$H''_\phi \approx \frac{1}{2\pi r} i(0,t-r/c) \tag{20}$$

According to equations (19) and (20), the lightning electric and magnetic field waveforms can be expressed approximately as the channel-base current waveform with a factor which is a function of the horizontal distance *r*. It is important to note that these equations are based on the TL model according to which the current propagates along the channel at a constant speed and without either distortion or attenuation.

Further, the time-derivatives of the lightning electromagnetic fields at near ranges can be obtained as:

$$\frac{\mathrm{d}E_z(r,t)}{\mathrm{d}t} \approx \frac{\mathrm{d}E'(r,t)}{\mathrm{d}t} \approx \frac{\mathrm{d}E''_z(r,t)}{\mathrm{d}t} \approx -\frac{1}{2\pi\varepsilon_0 vr}\frac{\mathrm{d}i\left(0,t-r/c\right)}{\mathrm{d}t} \tag{21}$$

$$\frac{\mathrm{d}H_\phi\left(r,t\right)}{\mathrm{d}t} \approx \frac{\mathrm{d}H'_\phi\left(r,t\right)}{\mathrm{d}t} \approx \frac{\mathrm{d}H''_\phi\left(r,t\right)}{\mathrm{d}t} \approx \frac{1}{2\pi r}\frac{\mathrm{d}i\left(0,t-r/c\right)}{\mathrm{d}t} \tag{22}$$

Evaluation of the approximate expressions for lightning electromagnetic fields at near ranges

In order to obtain the range of distances at which the approximate expressions are valid, the lightning channel-base current with an 8/20μs waveform is used to calculate the lightning electromagnetic fields. The length of the lightning channel is set to L=7.5km, and the return stroke speed is v=1.3×10^{8}m s^{-1}. Moreover, the pulse function [Zhang and Liu 2002] is chosen to describe the channel-base current, expressed as follows:

$$i(0,t) = \frac{I_0}{\eta}\left[1-\exp(-t/\tau_1)\right]^n \exp(-t/\tau_2) \tag{23}$$

where $\eta = [n\tau_2/(\tau_1 + n\tau_2)]^n [\tau_1/(\tau_1 + n\tau_2)]^{\tau_1/\tau_2}$. In this paper, we set n=2. The other parameters of the channel-base current are set as follows: $I_0 = 30$ kA, $\tau_1 = 4.0\times10^{-5}$ s, $\tau_2 = 6.25\times10^{-6}$ s. The electric and magnetic fields calculated from the approximate expressions, including the first-level approximation (equations (17) and (18)) and second-level approximation (equations (19) and (20)), along with the fields calculated from exact electromagnetic field expressions (equations (10) and (11)) at different distances from the lightning channel are shown in Fig. 2. Comparison between the electric and magnetic field derivative waveforms from the approximate expressions, including the first-level and second-level approximations, and the fields computed from exact expressions are given in Fig. 3.

As seen in Fig. 2, the waveforms predicted by the approximate expressions are similar to those based on the exact expressions. The deviations of the approximate results from the exact results increase with increasing distance r. The closer the distance, the less the deviation from the exact results. The approximate electric field waveforms are essentially coincident with the exact ones within 100 m. When the distance reaches 500 m, there is a significant difference between the approximate electric field waveforms and the exact ones. It can be seen in Fig. 2 that the difference between the exact magnetic field and its two approximations is smaller than that between the exact electric field and its two approximations at same distance. The magnetic field waveforms based on the two approximate expressions can hardly be distinguished from the exact results until the distance reaches about 500 m. It is important to note that the calculated electric field waveforms do not show the characteristic flattening after 15 μs or so (seen in measured waveforms), which is due to inadequacy of the TL model at later times [Rakov and Uman 1998].

As seen in Fig. 3, the deviations of the approximate electromagnetic fields derivative waveforms from the exact derivative waveforms also increase with the increasing distance r. The closer the distance, the less the deviation. The approximate electric field derivative waveforms are essentially coincident with the exact derivative waveforms within 100 m. When the distance reaches 500 m, there is a significant difference between the approximate and exact electric field derivative waveforms. From comparison of the waveforms of electric and magnetic field derivatives in Fig. 3, it can be seen that the difference between the exact magnetic field derivative and its two approximations is smaller than that between the exact electric field derivative and its approximation at the same distance. The approximate magnetic field derivative waveforms can hardly be distinguished from the exact ones until the distance reaches 500 m.

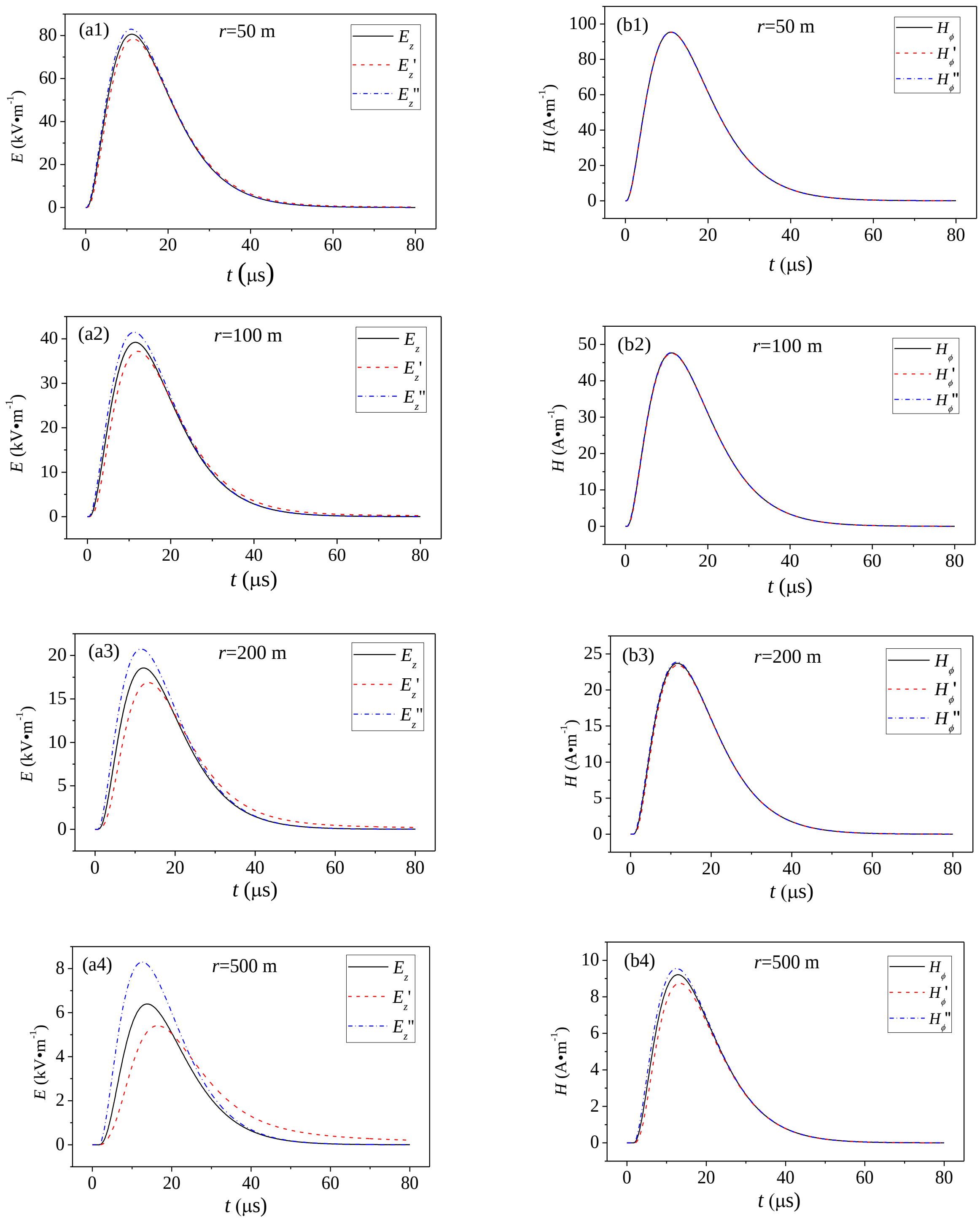

Fig. 2. (a) electric and (b) magnetic field waveforms predicted by the approximate expressions (the first-level and second-level approximations are marked by single and double primes, respectively) and those based on the exact expressions (unprimed field symbols) at different distances from the lightning channel.

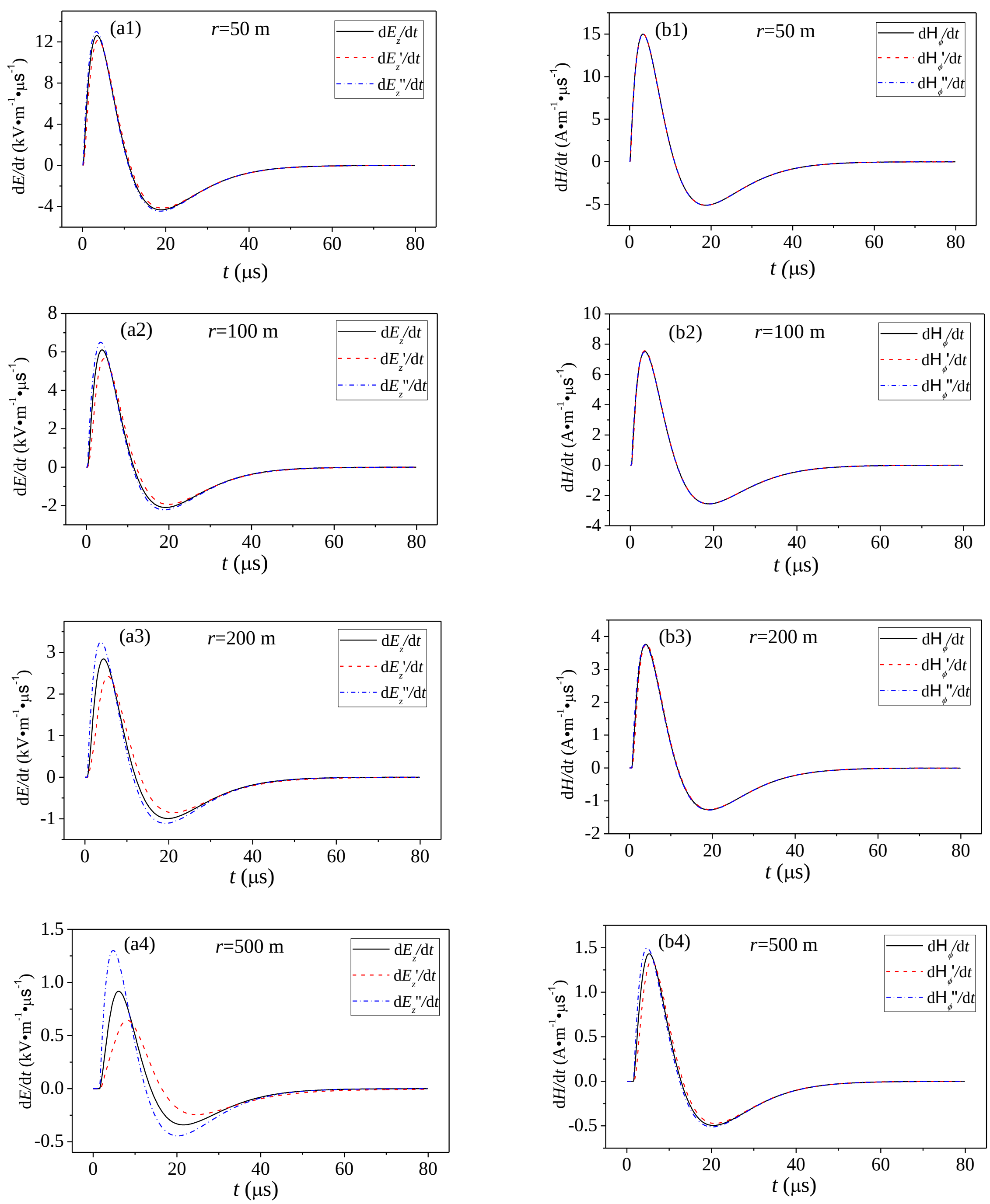

Fig. 3. (a) electric and (b) magnetic fields derivative waveforms predicted by the approximate expressions (the first-level and second-level approximations are marked by single and double primes, respectively) and those based on the exact expressions (unprimed field derivative symbols) at different distances from the lightning channel.

APPROXIMATE EXPRESSIONS FOR LIGHTNING ELECTROMAGNETIC FIELDS AND THEIR DERIVATIVES AT FAR RANGES

Approximate expressions for lightning electromagnetic fields at far ranges

At far ranges, $R \approx r$ and r >> L (L is the length of lightning channel), the radiation field is the dominant component of the electromagnetic field produced by lightning return stroke, the electrostatic and induction components can be neglected. We can simplify the expressions of electromagnetic fields as follows [Uman et al. 1975]:

$$E_z \approx E_z(\text{radiation}) \approx -\frac{1}{2\pi\varepsilon_0 c^2 r}\int_0^L \frac{\partial i(0, t - r/c - z'/v)}{\partial t}\,\mathrm{d}z' \tag{24}$$

$$H_\phi \approx H_\phi(\text{radiation}) \approx \frac{1}{2\pi c r}\int_0^L \frac{\partial i(0, t - r/c - z'/v)}{\partial t}\,\mathrm{d}z' \tag{25}$$

Since v is constant, we obtain:

$$\frac{\partial i(0, t - r/c - z'/v)}{\partial t} = -v\frac{\partial i(0, t - r/c - z'/v)}{\partial z'} \tag{26}$$

Thus, equations (24) and (25) can be written as:

$$E_z(\text{radiation}) \approx \frac{v}{2\pi\varepsilon_0 c^2 r}\int_0^L \frac{\partial i(0, t - r/c - z'/v)}{\partial z'}\,\mathrm{d}z' \tag{27}$$

$$H_\phi(\text{radiation}) \approx -\frac{v}{2\pi c r}\int_0^L \frac{\partial i(0, t - r/c - z'/v)}{\partial z'}\,\mathrm{d}z' \tag{28}$$

After integration, we get:

$$E_z(\text{radiation}) \approx -\frac{v}{2\pi\varepsilon_0 c^2 r}\times\left[i(0, t - r/c) - i(0, t - r/c - L/v)\right] \tag{29}$$

$$H_\phi(\text{radiation}) \approx \frac{v}{2\pi c r}\times\left[i(0, t - r/c) - i(0, t - r/c - L/v)\right] \tag{30}$$

When $\tau \le 0$, $i(0,\tau) = 0$. So, when $t \le L/v + r/c$ (the return-stroke front has not reached the channel top yet), equations (29) and (30) can be expressed as:

$$E_z(r,t) \approx E_z(\text{radiation}) \approx -\frac{v}{2\pi\varepsilon_0 c^2 r} i(0, t - r/c) \tag{31}$$

$$H_\phi(r,t) \approx H_\phi(\text{radiation}) \approx \frac{v}{2\pi c r} i(0, t - r/c) \tag{32}$$

From equations (31) and (32), the lightning electric and magnetic field waveforms can be expressed approximately by the channel-base current waveform with factors (different from those for near ranges) which are each a function of the return stroke speed v, the speed of light c, and the horizontal distance r, as long as the return stroke current has not reached the top of the channel. The negative sign in equation (31) means that the direction of the electric field is opposite to the direction of propagation of current wave (positive charge moving up).

The time derivatives of equations (31) and (32) are as follows:

$$\frac{\mathrm{d}E_z(r,t)}{\mathrm{d}t} \approx \frac{\mathrm{d}E_{\text{rad}}(r,t)}{\mathrm{d}t} \approx -\frac{v}{2\pi\varepsilon_0 c^2 r}\frac{\mathrm{d}i(0, t - r/c)}{\mathrm{d}t} \tag{33}$$

$$\frac{\mathrm{d}H_\phi(r,t)}{\mathrm{d}t} \approx \frac{dH_{\mathrm{rad}}(r,t)}{\mathrm{d}t} \approx \frac{v}{2\pi cr}\frac{\mathrm{d}i(0,t-r/c)}{\mathrm{d}t} \tag{34}$$

Evaluation of the approximate expressions for lightning electromagnetic fields at far ranges

Equations (31) and (32) can be used for calculation of electromagnetic field only when the horizontal distance *r* is much larger than the channel height L. However, the rise time to the peak of the lightning electric and magnetic fields is only some microseconds or less. The field derivative corresponding to the wavefront is much larger than that corresponding to the tail. Thus, the effect of the electromagnetic field derivatives in the initial several microseconds or less will be dominant.

In order to validate the approximate expressions at far ranges, the lightning channel-base current with an 8/20 μs waveform was adopted to calculate lightning electromagnetic fields. Accordingly, the parameters of the channel-base current were set as follows: $I_0 = 30\,\mathrm{kA}, \tau_1 = 4.0\times10^{-5}\,\mathrm{s}$, $\tau_2 = 6.25\times10^{-6}$ s. The lightning channel length was set to *L*=7.5 km, and the return stroke speed to $v = 1.3\times10^8$ m/s. The waveforms of the radiation fields, the scaled channel-base current and the exact electromagnetic fields at distances 5 km and beyond are given in Fig. 4. The waveforms of radiation field derivatives, the scaled channel-base current derivatives and the exact electromagnetic fields derivatives at different distances are shown in Fig. 5. Here, two different factors are introduced to be used with the scaled channel-base current. For the electric field and its derivative, the multiplying factor for the current and its derivative is q_1; while for the magnetic field and its derivative, the multiplying factor for the current and its derivative is q_2. According to equations (31)-(34), the values of q_1 and q_2 are set as follows:

$$q_1 = \frac{v}{2\pi\varepsilon_0 c^2 r}, \quad q_2 = \frac{v}{2\pi cr}$$

As follows from Fig. 4, the differences among the waveforms of the radiation fields, the scaled channel-base currents, and the exact electromagnetic fields reduce when the distance r increases. This is not surprising, since at 5 and 10 km significant contributions from electrostatic and induction field components (not accounted for in the approximate expressions) are present. The smaller the distance, the greater the differences. However, the rising portions of the radiation field and scaled channel-base current waveforms are essentially coincident with those of the exact electromagnetic field waveforms. The deviations between the radiation field waveforms and the scaled channel-base current waveforms are relatively small. These deviations are mainly caused by the approximate substitution of r for R in equations (31) and (32). As seen in Figs. 4 (a3) and (b3), at a distance of 50 km, the risetimes of the exact electromagnetic field, radiation field, and scaled channel-base current are nearly the same. There is still some difference among the waveforms of the exact electromagnetic fields, the radiation fields, and the scaled channel-base currents, because the distance of 50 km is only about 7 times larger than the length of the channel, which can not be viewed as *r* >> *L* (the far field approximation condition is not satisfied perfectly). When the distance is 100 km, as seen in Figs. 4 (a4) and (b4), the waveforms of radiation field are essentially coincident with the scaled channel-base current waveforms. There is only slight difference, which occurs at late times, between the waveforms of the total electromagnetic fields and the radiation fields. It is important to note that the waveforms at 50 and 100 km do not show the characteristic zero-crossing (seen in measured waveforms), which is due to inadequacy of the TL model at later times

[Rakov and Uman 1998].

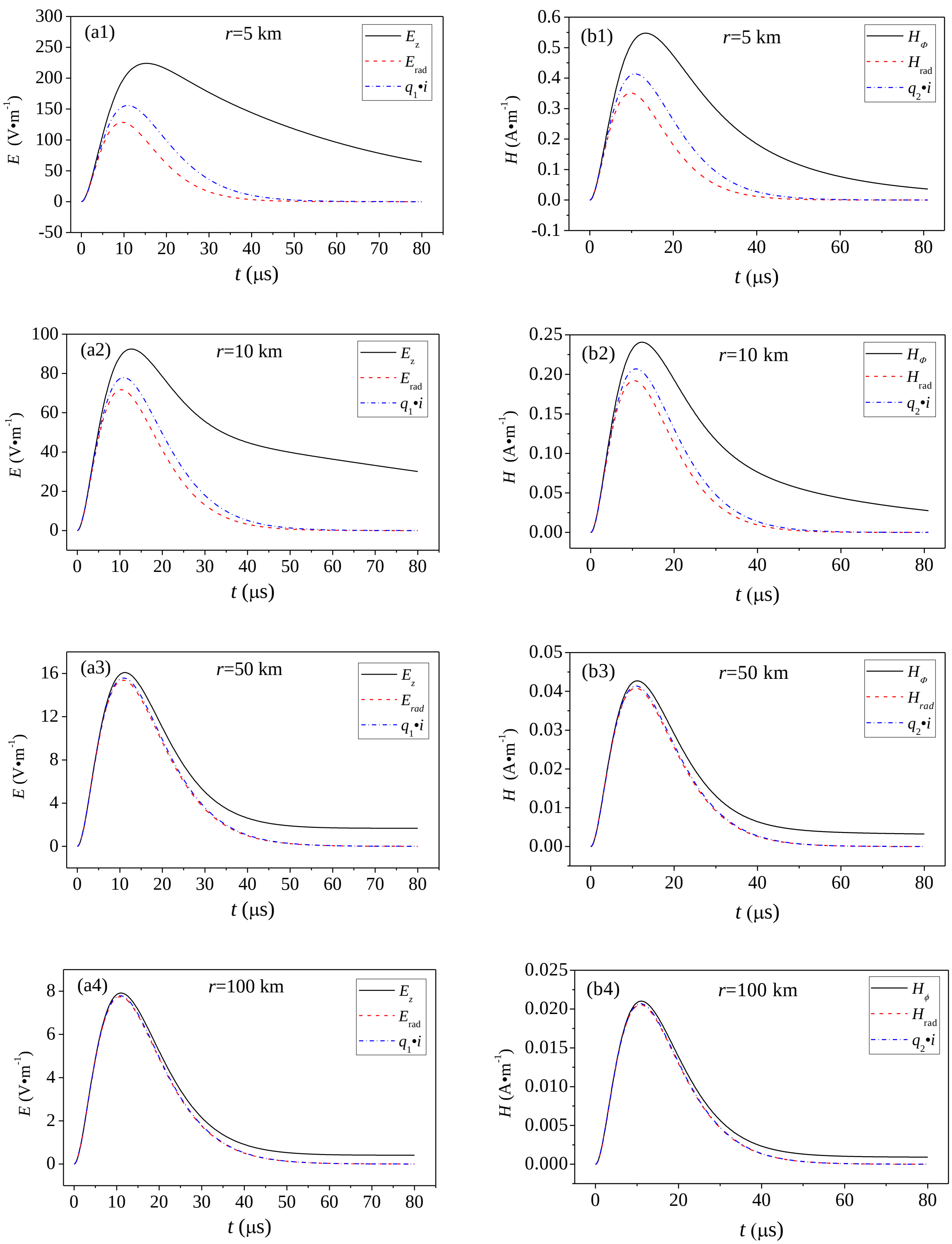

Fig. 4. (a) electric and (b) magnetic field waveforms predicted by the radiation field expressions and those based on the exact expressions (including all field components) at different distances from the lightning channel. Also shown in each plot is the corresponding scaled current (approximate far-field equation introduced here).

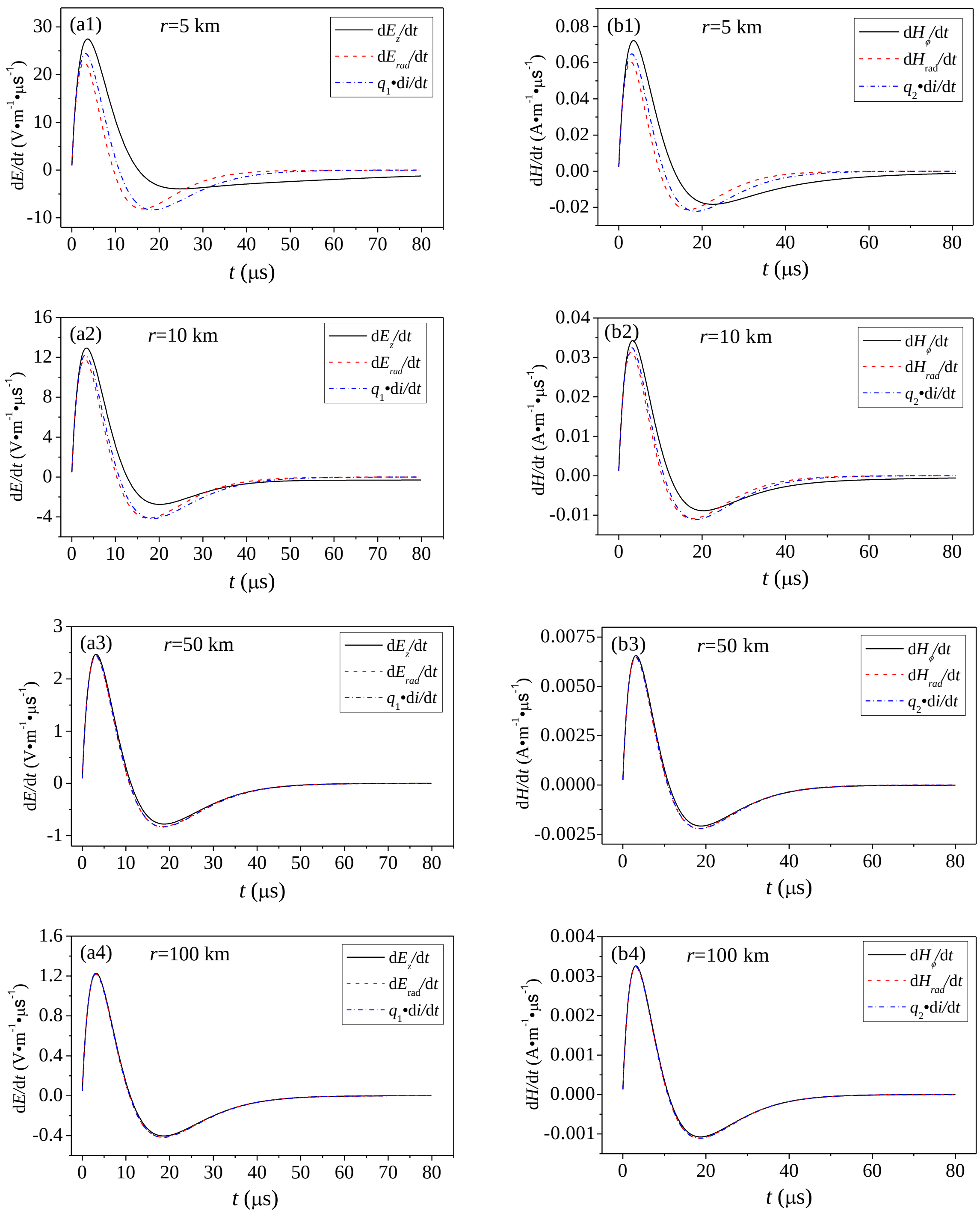

Fig. 5. (a) electric and (b) magnetic field derivative waveforms predicted by the radiation field expressions and those based on the exact expressions at different distances from the lightning channel. Also shown in each plot is the corresponding scaled current (approximate far field equation introduced here).

As seen in Fig. 5, the deviations between the approximate results and exact result decrease with increasing r. The closer the distance, the greater the deviations. The time derivative waveforms of the radiation field and the scaled channel-base current are basically coincident with those of the total (exact) electromagnetic field in the initial several microseconds at distances beyond 5 km. This is because the height the return stroke current arrives at in the initial several microseconds is generally several hundreds of meters. Thus, at distances beyond 5 km, the far field approximation condition is satisfied.

Comparing Fig. 4 with Fig. 5, we can see that the differences among the exact field derivatives, the radiation field derivatives, and the scaled channel-base current derivatives are smaller than the differences among the exact fields, the radiation fields, and the scaled channel-base currents at the same distance. It can be concluded that the approximation condition for the electromagnetic fields at far ranges is more strict than that for their derivatives. The approximate expressions for electromagnetic field derivatives are valid at distances beyond several kilometers, and the approximate expressions for electromagnetic fields are valid beyond 50 km.

SUMMARY AND CONCLUDING REMARKS

Based on the TL model, the approximate expressions for the lightning electromagnetic field on the earth surface at near and far ranges are obtained. The approximate expressions show that the electric and magnetic fields waveforms can be expressed approximately by the channel-base current waveform with a factor which depends on distances. The validity of the approximate expressions is analyzed by comparing the waveforms calculated from the approximate expressions at different ranges with the corresponding exact expressions. The results show that the ranges at which the approximate expressions are valid at near ranges are within 100 m, 100 m, 500 m, and 500 m for the electric field, electric field derivative, magnetic field, and magnetic field derivative, respectively. The ranges at which the approximate expressions are valid at far ranges are beyond 50 km, 5 km, 50 km, and 5 km for the electric field, electric field derivative, magnetic field, and magnetic field derivative, respectively. If the return stroke speed is increased, the ranges will be extended. In the limit, if the return stroke speed were equal to the light speed c, the waveforms of electromagnetic fields will be same as the waveform of the lightning channel-base current at any distance from the lightning channel, as shown theoretically for the TL model by Thottappillil et al. [2001].

It should be pointed out that there is no hump in the computed electromagnetic fields at near ranges although it is often seen in the measured fields. However, it does not matter when only the rise time and the initial peak are needed.

ACKNOWLEDGMENTS

This research was supported by National Natural Science Foundation Project of China under Grant No. 51377171 and by the NIMBUS Program.

REFERENCES

Barbosa, C. F., Paulino, J. O. S., and de Miranda, G. C. *et al*., 2008: Measured and modeled horizontal electric field from rocket-triggered lightning, *IEEE Trans. Electromagn. Compat*., vol. 50, no. 4, pp. 913-920.

Cooray, V., 1993: A model for subsequent return strokes, *J. Electrostat*., vol. 30, pp. 343-354.

Cooray, V., Fernando, M., and Gomes, C. *et al*., 2004: The fine structure of positive lightning return-stroke radiation fields, *IEEE Trans. Electromagn. Compat.*, vol. 46, no. 1, pp. 87-95.

Hoole , P. R. P., and Balasuriya , B. A. A. P.. 1993: Lightning radiated electromagnetic fields and high voltage test specifications, *IEEE Trans. Magnetics*, vol. 29, no. 2, pp. 1845-1848.

Hussein, A. M., Milewski, M., and Janischewskyj, W., 2008: Correlating the characteristics of the CN tower lightning return-stroke current with those of its generated electromagnetic pulse, *IEEE Trans. Electromagn. Compat*., vol. 50, no. 3, pp. 642-650.

Jerauld, J., Uman, M. A., and Rakov, V. A., 2007: Insights into the ground attachment process of natural lightning gained from an unusual triggered-lightning stroke, *J. Geophys. Res*., vol. 112, pp. D13113-1-D13113-16.

Jerauld, J., Uman, M. A., and Rakov, V. A. *et al*., 2008: Electric and magnetic fields and field derivatives from lightning stepped leaders and first return strokes measured at distances from 100 to 1000 m, *J. Geophys. Res*., vol. 113, no. D17111, doi: 10.1029/2008JD010171.

Jerauld, J. E., Uman, M. A., and Rakov, V. A. *et al*., 2009: Measured electric and magnetic fields from an unusual cloud-to-ground lightning flash containing two positive strokes followed by four negative strokes, *J. Geophys. Res*., vol. 114, no. D19115, doi:10.1029/2008JD011660.

Lee, B. H., Eom, J. H., and Kang, S. M. *et al*., 2004: Characteristics of the radiation field waveforms produced by lightning return strokes, *Jpn. J. Appl. Phys*., vol. 43, pp. 4379-4385.

Lin, Y. T., Uman, M. A., and Tiller, J. A. *et al*., 1979: Characterization of lightning return stroke electric and magnetic fields from simultaneous two-station measurements, *J. Geophy. Res*., vol. 84, pp. 6307-6314.

Miyazaki, T., Ishii, T., and Okabe, S., 2010: A field study of lightning surges propagating into residences, *IEEE Trans. Electromagn. Compat*., vol. 52, no. 4, pp. 921-928.

Nucci, C. A., Diendorfer, G., and Uman, M. A. *et al*., 1990: Lightning return stroke current models with specified channel-base current: A review and comparison, *J. Geophys. Res*., vol. 95, pp. 20395-20408.

Rakov, V. A., and Uman, M. A., 1998: Review and evaluation of lightning return stroke models including some aspects of their application, *IEEE Trans. Electromagn. Compat*., vol. 40, pp. 403-426.

Rakov, V. A., and Uman, M. A., 2003: *Lightning: Physics and Effects*, Cambridge.

Rakov, V. A., and Rachidi, F., 2009: Overview of recent progress in lightning research and lightning protection, *IEEE Trans. Electromagn. Compat*., vol. 51, no. 3, pp. 428-442.

Rubinstein, M., and Uman, M. A., 1989: Methods for calculating the electromagnetic fields from a known source distribution: application to lightning, *IEEE Trans. Electromagn. Compat*., vol. 31, pp. 183-189.

Samaras, K., Sandberg, C., and Salmas, C. J. *et al*., 2007: Electrical surge-protection devices for industrial facilities—A tutorial review, *IEEE Trans. Ind. Applicat*., vol. 43, no. 1, pp. 150-161.

Stratton, J. A, 1941: *Electromagnetic Theory*, McGraw-Hill, New York, USA

Thottappillil, R., Schoene, J., and Uman, M. A., 2001: Return stroke transmission line model for stroke speed near and equal that of light, *J. Geophys. Res. Lett*., vol. 28, pp. 3593-3596.

Thottappillil, R., 2002: Electromagnetic pulse environment of cloud-to-ground lightning for EMC studies, *IEEE Trans. Electromagn. Compat*., vol. 44, no. 1, pp. 203-213.

Uman, M. A., and McLain, D. K., 1969: Magnetic field of the lightning return stroke, *J. Geophys. Res*., vol. 74, pp. 6899-6910.

Uman, M. A., Mclain, D. K., and Krider, E. P., 1975: The electromagnetic radiation from a finite antenna, *Amer. J. Phys*., vol. 43, pp. 33-38.

Uman, M. A., Rakov, V. A., and Schnetzer, G. H. *et al*., 2000: Time derivative of the electric field 10, 14, and 30 m from triggered lightning strokes, *J. Geophys. Res*., vol. 105, no. D12, pp. 15577-15595.

Uman, M. A., Schoene, J., and Rakov, V. A.,2002: Correlated time derivatives of current, electric field intensity, and magnetic flux density for triggered lightning at 15 m, *J. Geophys. Res*., vol. 107, no. D13, ACL1-1-ACL1-11.

Zhang, F. Z., and Liu, S. H., 2002: A new function to represent the lightning return-stroke currents, *IEEE Trans. Electromagn. Compat.*, vol. 44, no. 4, pp. 595-597.