J3.6 A HAZARD MODEL FOR TORNADO OCCURRENCE IN THE UNITED STATES

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1. Introduction

The development of techniques to assess hazard probabilities will greatly benefit many people throughout the United States. As population increases, the damage due to a severe event such as a tornado also increases. Through the use of a hazard model, tornado occurrence, location, path length, path width, and F-scale can be modeled. With the knowledge of tornado occurrence and intensity, individuals and businesses can better prepare for a significant event (tornadoes F2 or greater on the Fujita scale). Areas with large populations will be able to develop better plans for tornado events. The risk for a specific date and location can be determined, and with this information damages and injuries can be reduced.

Without a hazard model, there are only about 75 years of useful data to use to determine the climatology of tornadoes, since the database of tornado reports in Grazulis (1993) only has a significant number of records from the early 1920's. Especially within the past few decades, more and more tornadoes have been reported due to the improvement of technology and public awareness of severe weather. This gives a larger dataset, however it may still not be large enough to analyze on its own when it is taken into consideration that tornadoes might be acting on a pattern thousands of years in length. Tornadoes are rare events everywhere; to see large-scale variability, a sample size of thousands of years must be used.

Earlier work has been done in hazard modeling by Schaefer et al. (1986) (hereafter SKA). In this assessment, a minimum assumption model is used. Each tornado report is looked at from 1950-1983, and the areal coverage is determined from the lengths and widths of each report. SKA, using overlapping Marsden squares, take each section of the United States as being independent from the rest, and all of the tornado information is specific for that location. By using this method, spatial variability throughout the United States can be shown in detail.

In order to build our hazard model, the Grazulis dataset (1921-1995) is used. These seventy-five years give a good sampling of tornadoes, with approximately 10,000 tornadoes with damage intensity, path length and width data. Intensities are given by the Fujita scale (Fujita 1971), with damage increasing from F0 to F5. We look only at strong and violent tornadoes, which includes those classified F2 and greater. The seventy-five years of data are analyzed and fit to statistical distributions so that many years can be generated from the model. In this run, thirty thousand years of data are produced. With this model, long-term patterns can be observed and variability between different time periods can be analyzed. Knowing how significant tornado events vary from year to year is extremely helpful in determining the overall risk. Using the variability and trends from the model, the risk of a significant tornado can be seen for any given location. With the output from this model, future changes in the climatology of tornado occurrence might be foreseen.

2. Approach

a. Monte Carlo Model

In order to model significant tornado occurrences in the United States, a Monte Carlo method is used. In the long run, this model will match the statistical distributions that it is run off of. Each time it runs, the model takes in a random number, and using that number as a seed it pulls all of the tornado data from the same statistical fits each time. The model output exhibits reasonably good variability, resembling the raw data. This is an amazing thought when it is taken into account that there are no physical processes going into the model, it is based entirely on statistics, not on the atmosphere. The model looks at the entire United States with a grid having grid boxes approximately 80 km on a side. Each day of the year and each grid box are considered independent for the most part. The only dependencies between locations involves a random parameter that determines whether a particular year is a "good" or "bad" tornado year, and a second random parameter that determines whether a particular day is a "good" or "bad" tornado day. The former is an engineering fix to inflate the variance of the number of tornadoes per year by about 20% after the initial "uninflated" model was found to have too small of a variance. The latter is designed to allow the model to mimic

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tornado outbreaks on a particular day, but it has no effect on the statistics we will look at in this paper.

By using a statistical distribution to describe the tornado characteristics, thousands of numbers can be described by knowing only five characteristics. The first is the probability of a tornado occurring. This is dependent upon the location that is being looked at; there are higher probabilities in areas where relatively high numbers of tornadoes occur. The probability that a tornado will occur on any one day is fit by a Beta distribution (McCormick 2000) fit to the observed distribution of probabilities from Concannon et al. (2000). The second is the number of tornadoes that occur on a specific day, given that a tornado will occur. This is taken from an empirical fit to the observed number of tornadoes on days that tornadoes occurred from 1921-1995. Third, the F-scale must be determined based on the probability distribution functions of each rating in the raw F-scale statistics (Brooks and Doswell 2001). As a simple model for this, we have assumed that there are 0.3 times as many F3 tornadoes as F2, 0.2 times as many F4 tornadoes as F3 tornadoes, and 0.1 times as many F5 tornadoes as F4. The fourth and fifth elements are the length and width of the tornado's path, which are both taken from Weibull distribution fits. A parameter is also included in the model to inflate the variance of the number of tornadoes per year without affecting the mean. It is essentially a random number that determines whether a year has a large number of tornadoes or not.

b. Statistical Fits to Length and Width

Concannon et al. (2000) and McCormick (2000) developed a model for the occurrence of tornadoes on any day at any location in the United States. In order to turn this model into a model of the tornado hazard, we need to include information on path length and width. It is assumed that length, width, and intensity are independent of location, and random samples are taken from the underlying distributions that describe them. Statistical distributions feed the model; individual tornadoes are not looked at. By estimating the statistical distributions from the entire country, we have a larger sample size to look at sample variability. Regional analysis of the distribution of tornadoes by intensity have shown that, for the region east of the Rocky Mountains, the distribution is essentially constant for F2 and greater tornadoes outside of Florida (Brooks and Doswell 2001). Analysis of the regional variability of length and width has shown no clear patterns, so we have chosen to assume that the distribution is constant over the country. It is possible that there are real spatial differences, but the dataset is inadequate to detect them with any statistical confidence.

In order to incorporate the path length and width data into the Monte Carlo model efficiently, we tested a variety of distributions. In looking for a distribution to fit the lengths and widths, several criteria have to be met. The curve has to be positively skewed, and it must be nonnegative. Ideally, it would have a defined and analytically integrable cumulative distribution function (CDF) in order to facilitate model calculations.

The Weibull distribution is found to be the best fit to the path lengths and widths. In using the Weibull distribution, only two parameters are needed to describe the curve, α and β. α describes the shape of the distribution, or where it peaks on the x-axis. When a equals one, the curve reduces to the exponential function, intersecting the y-axis at its peak. If $\alpha < 1$, the curve resembles a backwards 'J' shape and becomes more positively skewed. For $\alpha > 1$, the curve moves away from the y-axis and becomes more sharply peaked. The second parameter, β, describes the scale, or "stretch" of the curve. For a given value of α, β works to either stretch or compress the curve along the x-axis. The formula for the Weibull CDF is $F(x) = 1 - \exp[-(x/\beta)\alpha]$. The distribution mean is given by $\mu = \beta\Gamma(1+1/a)$ and the variance is $\sigma^2 = \beta^2[\Gamma(1 + 2/\alpha) - \Gamma^2(1 + 1/\alpha)]$. In most of our cases, α is approximately 1.0, and the mean simplifies to β and the variance simplifies to β^2.

Qualitatively, the Weibull distributions fit the data well, with the fits being best at F2. Tornadoes become wider as the F-scale increases, with the median F2 being about 100 m wide and the median F5 being 600 m wide.(Fig. 1) Similarly, tornado path length increases with F-scale, with the median increasing from about 10 km for F2 to 60 km for F5 (Fig. 2).

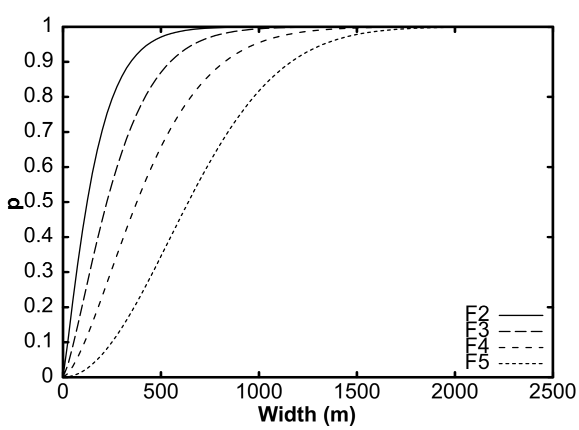

Figure 1: CDF of Weibull fits to width data by F-scale for all tornadoes, 1921-1995.

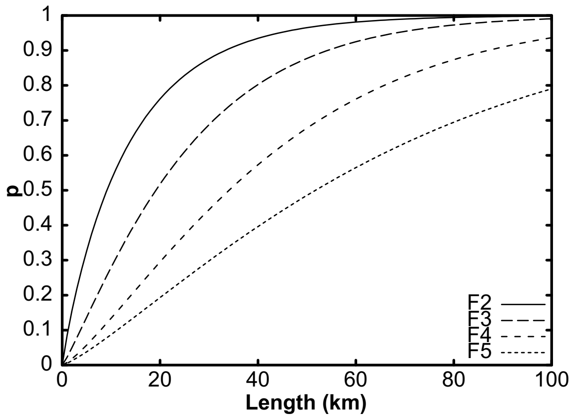

Figure 2: Same as Fig. 1 except for length data.

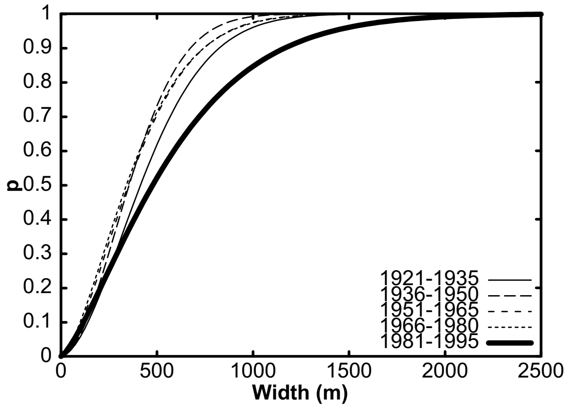

Figure 3: CDF of Weibull fits to width data for F4 tornadoes by 15-year periods. 1981-1995 in bold.

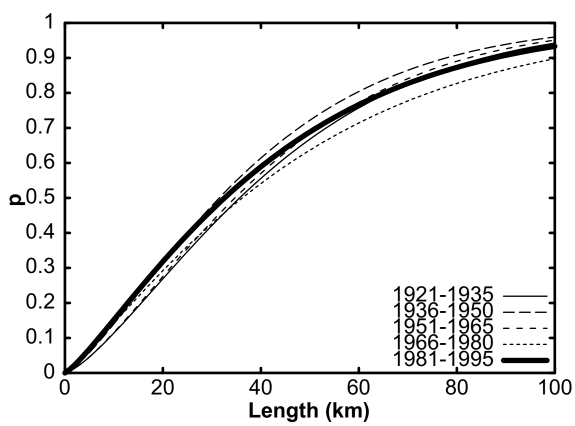

Figure 4: Same as Fig. 3 except for length data.

Variability of these estimates during the period of record is another issue of importance. Although we need to exercise caution, particularly at the high end as sample sizes get smaller (there are about 10 F4 tornadoes per year in the United States and 1 F5 tornado per year), periods of 15 years give reasonable sample sizes while still allowing us to look at temporal changes for everything except the F5 tornadoes. The biggest change in the record is seen in the widths of F4 tornadoes, particularly for the widest tornadoes (Fig. 3). Even though the lowest quartile of the distribution is about the same, the upper half of the data clearly show a tendency for wider tornadoes in the last 15 years of the record. This likely results in a change in the instructions given to the people within individual National Weather Service offices who collect the reports to change from reporting average widths to reporting maximum widths (J. T. Schaefer, personal communication), although it is difficult to determine operationally how sufficient data to determine an "average" width has been obtained historically. The length data, on the other hand (Fig. 4), show no such tendency for significant temporal changes.

Even without running the Monte Carlo model, the distributions of observations obviously illustrate important aspects of tornado reports. The fact that the data can be fit well to simple statistical distributions suggests that we can model the occurrence can provide insight into the variability of tornado occurrence.

3. Results

The Monte Carlo model generated 30,000 years of tornado occurrences, including almost four million individual tornadoes. We have resampled with replacement the 75 observed years 30,000 times in order to facilitate comparisons. In what follows, this dataset will be referred to as the "resampled observations" and the 75 years will be referred to as the "raw observations". In addition, we have taken means of non-overlappling 15-year periods of both the Monte Carlo and resampled datasets in order to get some feel for what possible variability we might be able to see in the raw observations. This gives a total of 2000 "subperiods."

This abundance of information first has to be checked to see how well it fits reality. In general, although the average value of the model parameters is reasonably good, being slightly low, the model has less variability than the observations. The median number of tornadoes per year in the observed data is 131, while it is only 128 from the model. The interquartile range of the 15-year means is 11.0 for the Monte Carlo model and 13.7 for the resampled dataset.

a. Variability of the results

A similar result is seen when length and width data are examined. Quantile-quantile plots of the length (Fig. 5) and width (Fig. 6) for F2 tornadoes show that the tails of the model distributions are too light. This problem is somewhat worse for the F5 tornadoes, in large part as a result of the smaller sample size. We can also see that the fitting problem is more challenging for the width data than for the length data. This is exacerabated by the “quantization” of the observations. For example, there are many reports of tornadoes that are 100 yards (raw observations are in English units) wide, but none that are 110 yards wide. In general, most tornado lengths more than a few miles long are reported to the integer mile. For long paths, i.e., the more intense tornadoes, this quantization doesn’t cause too much difficulty, although the problem of sample size remains. For the width data, however, quantization is a serious problem throughout the range of values. Reports of width are almost exclusively either in hundreds of yards or in simple fractions (quarter, half) of a mile. This makes fitting the distributions difficult and we believe the small discrepancies seen in Figs. 5 and 6 are not particularly serious. In fact, it is possible that, given the problem of quantization, that the model may match reality better than the reports.

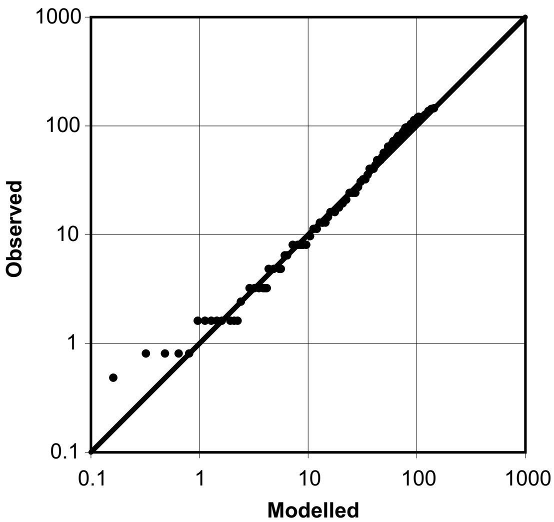

Figure 5: Quantile-quantile plot of observed and modelled F2 tornado path length in km. Note logarithmic scale.

The length and width from the model provides the input for calculating the area of each tornado. We have assumed that all of the area associated with a particular model tornado occurs at the grid box in the model that tornado touches down in. Clearly, some of the paths are longer than 80 km, so that the real-world equivalent tornadoes would cross grid boxes. For computational simplicity, we have ignored this issue. In reality, this effect should be balanced at a particular grid point, except for those on the edges of the domain, by neglecting

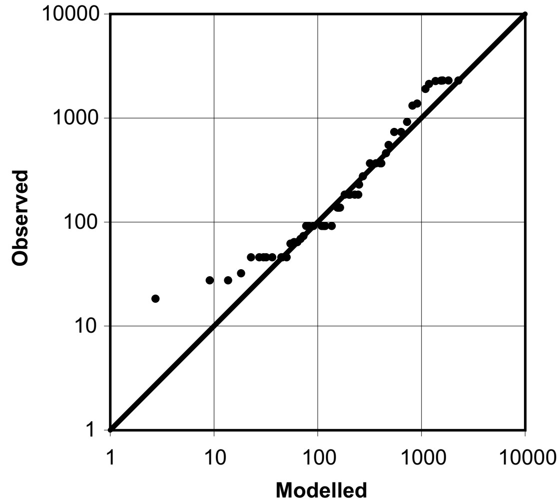

Figure 6: Quantile-quantile plot of observed and modelled F2 tornado path width in m. Note logarithmic scale.

what would be coming in from adjacent grid boxes. In addition to that, we have neglected the direction of tornado movement. Data on that is sparser than on the other aspects of the dataset and neglects changes in the direction that may occur during the time a tornado is on the ground. It is doubtful that this is a major effect and, because the data on it is so sparse, it is unlikely that we could verify any aspect of model performance using it.

The underestimate of the variability of the

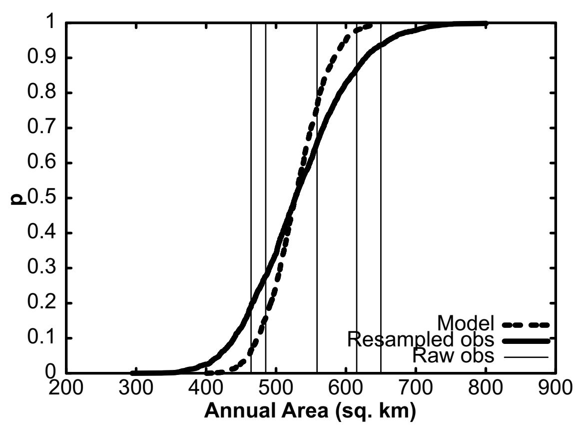

Figure 7: CDF of mean annual areal coverage, in square kilometers, of F2 or greater tornadoes based on 15-year subperiods. Dashed line from Monte Carlo simulation, thick solid line from resampled observations, and thin vertical lines from non-overlapping periods in raw observations.

length and width shows up in the areal coverage. CDFs constructed from the mean areal coverage over the United States based on 15-year periods show that the model is slightly biased below the resampled observations, but clearly has too small of a variance (Fig. 7). The 10th and 90th percentiles of the Monte Carlo model correspond roughly to the 25th and 75th percentiles of the resampled observations. Furthermore, comparison of the resampled observations to the raw observations demonstrates that non-overlapping 15-year subperiods of the 75-year observations have been insufficient to estimate the actual variability implicit in the observations. None of the periods in the observations have been between the 35th and 65th percentiles of the resampled observations.

The spatial distribution of the coverage associated with the variability is of interest as well. If we assume that tornadoes occurring on different days are independent events, it is obviously reasonable that *somewhere* in the United States will get more than one significant tornado in its vicinity in a relatively short period of time. From the societal standpoint, however, since tornadoes are rare events at any particular location, people who happen to live near that location may interpret the events as something other than random! This issue becomes of more importance when we consider that decisions about threats or resource allocation have often been made based on short data records or, even individual events. it is also of importance because of the question of whether we can identify possible signals in tornado occurrence as a result of global climate change (e.g., IPCC 2001).

One way of illustrating this is to look at the minimum and maximum mean areal coverage of tornadoes for non-overlapping 15-year periods from the Monte Carlo simulation. Because the model has too little variability, the extremes of the model correspond roughly to the 5th and 95th percentiles of the resampled observations (Fig. 7). As a result, it's reasonable to use them as proxies for "big" and "small" periods. For simplicity, we will use the percentage of area within each grid box that would be hit by an F2 or greater tornado per millenium, based on the 15-year period, as a convenient unit. The maximum areal coverage (Fig. 8) shows a general area of high values over the southern United States, with peak values reaching almost 50% of a grid box in southern Mississippi. In comparison, the minimum areal coverage (Fig. 9) still has a region of high coverage over the southern United States, but the actual values within that region are very different. Southern Mississippi has appoximately half of the coverage in the minimum period that it has in the maximum period. The southeastern Oklahoma maximum decreases from over 35% to less than 25%. Other details include the presence of a maximum over Iowa in the largest coverage period that corresponds to a region that has a minimum in the smallest coverage

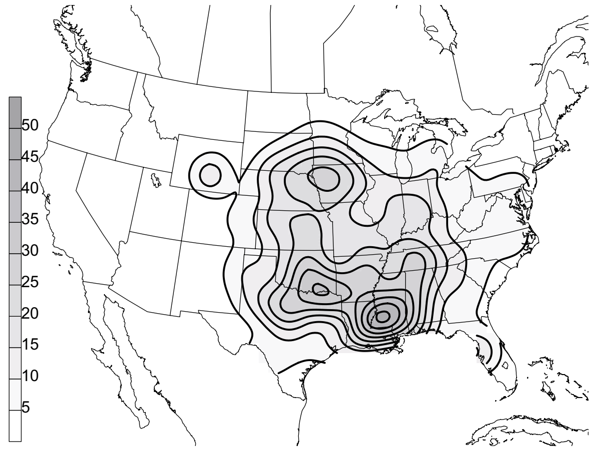

Figure 8: Percent areal coverage of grid boxes for largest coverage 15-year period from Monte Carlo simulation.

period. Note that southern Virginia actually has greater coverage of tornadoes in the period associated with the national minimum coverage than it does in the period with the national maximum coverage.

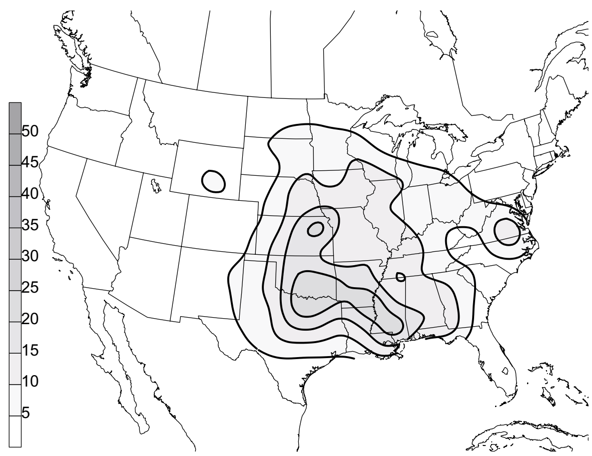

Figure 9: Same as Fig. 8, except for smallest coverage period.

The underlying statistical model here has no physical inputs. Changes in the occurrence and distribution of tornadoes would have to be on the order seen here from the model, if we hope to detect real changes in distribution of physical properties associated with tornado occurrence with the current record. This seems to be a daunting task given the very large variability implied by Figs. 8 and 9.

b. Return period of tornadoes

A common way of expressing the frequency of high-impact, rare events such as tornadoes is to plot a map of the return period (e.g., SKA). The return period can be thought of as the mean time between occurrences of an event given an infinitely long record. For the tornado coverage problem, this is simply a matter of taking the reciprocal of the coverage shown in Figs. 8 and 9, for example. An areal coverage of 25% (10%) per millenium would be associated with a return period of 4000 (10,000) years at that location.

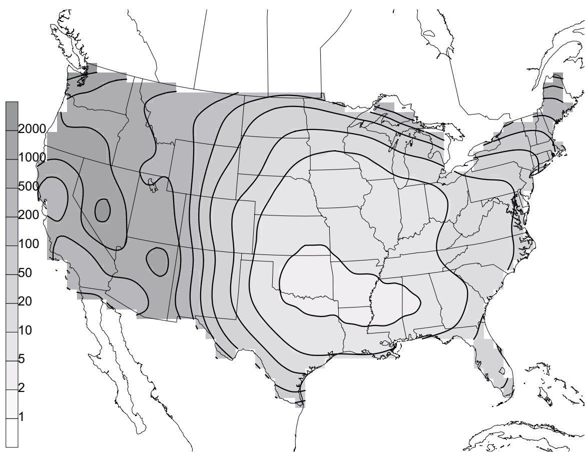

Figure 10: Return period for F2 or greater tornadoes in thousands of years from Monte Carlo simulation. Note that contours follow progression 1, 2, 5, 10,etc.

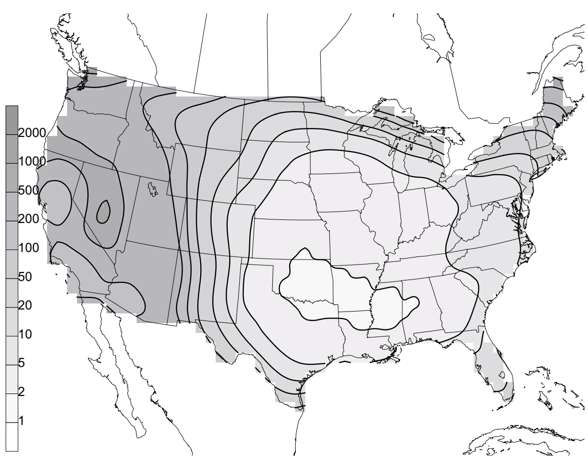

Figure 11: Same as Fig. 10, except for F4 or greater tornadoes in tens of thousands of years.

Using the entire 30,000 year Monte Carlo simulation to smooth out the variability seen in Figs. 8 and 9, we find that the minimum return period for an F2 or greater tornado at any location in the United States is approximately 4000 years (Fig. 10). A large part of the central United States has a return period of less than 10,000 years. The maximum return period, located over central Nevada, is greater than 5,000,000 years. Some caution must be attached to the exact values in the West, given that the assumptions that the distribution of tornadoes by F-scale and the distribution of lengths and widths are constant at all locations are unlikely to be met there. In addition, the number of events there that went into the development of the beta probability distributions is small, so that the reliability of the model there is suspect. Given those caveats, the values from the Rockies westward (with the possible exception of the valleys in California where there are more events in the dataset) are probably lower bounds.

Looking at more intense, rarer events, such as the F4 or greater tornadoes, the model produces a similar pattern, although the values for the return period are necessarily larger (Fig. 11). The minimum return period for an F4 or greater tornado is 16,700 years, while the maximum generated by the model is almost 18 million years. The differences in details of the F2 and F4 return period patterns are due to changes in the sample size from the model. Since the parameters of the individual tornadoes are identical, other than probability of occurrence, there can be no other explanation.

In comparing the hazard maps from SKA, with the maps from running the Monte Carlo model (Figs. 10 and 11), it can be seen that there are many similarities between them. (The two sets of maps are displayed differently, with the SKA F2 or greater map showing the log of the probability of annual occurrence.) The two F2 maps show a lot of agreement; both have a large area in the central United States with return periods of less than 10,000 years, with an absolute minimum in the return period over Oklahoma. They also show the very strong gradient west of the Plains States. The SKA map shows more detail in the western portion of the United States; this might be due to the lighter smoother that is applied by using overlapping Marsden squares in their hazard model. In the Monte Carlo model, a Gaussian smoother with σ=120 kilometers in each direction is used, allowing only the more general trends and maxima to be shown.

Upon examining the F4 maps, it can be seen that there is less agreement between them than there is between the F2 maps. There are some minima in the western portion of the United States that are not included in the SKA F4 map, whereas this map also has more maxima in the eastern portion of the U.S. Since F4's and F5's occur less often than other tornadoes, the grid is not as evenly distributed and it would require more smoothing to make the two maps look more alike. The overall detail in the SKA maps might be due to the small sample size that was used. Having only the observed few thousand tornadoes over the entire United States to sample from creates a less evenly distributed grid, whereas having millions of tornadoes, as in the Monte Carlo model, helps to portray more tornadoes occurring throughout the entire United States. The underlying question becomes, as with all statistical models, how well the distributions represent “truth.”

In the two Monte Carlo model maps, the contours are very similar. The return probabilities are obviously much smaller in the F4 map, but the lines trace the same general path with only a few minor differences. There is still a minimum in central Nevada and a maximum in Oklahoma, extending eastward into Alabama. The similarities in these two maps suggest that tornado occurrence is fairly constant in certain areas of the United States. In central Oklahoma and through Arkansas a lot of tornadoes occur, and so that is where the maxima for all tornadoes will be, regardless of the F-scale.

In comparing the two SKA maps, it can be seen that there aren’t as many similarities between them. The entire western portion of the United States is not even contoured in the F4 map. The lines extend differently throughout Texas, and in Nebraska there is a maximum for F4 occurrence very close to where there is a minimum for F2 occurrence. There is also a minimum in Arkansas for F4’s, but a maximum for F2’s. This suggests that the data used to create these maps may not complete, or that it is not taken over a long enough period of time to get a good handle on tornado occurrence throughout the United States.

These figures show that, by using a Monte Carlo model, although the results still agree to an extent with previous work in modeling tornado hazards, they are smoother and more evenly distributed across the entire United States. One possible explanation for this is that a very large sample time is needed in order to see definite trends and variability.

4. Discussion

Through the use of a hazard model, the risk of a significant tornado event across the United States can be determined. The Monte Carlo model produced results that looked realistic when compared to the raw data. There is a high risk in central Oklahoma and the surrounding area, extending eastward into Alabama, with a low risk in the far northeastern and western portions of the country. The fact that the model output matches the raw data so well is evidence that the Weibull distribution is a good fit to the path lengths and widths. If the Weibull distribution did not fit the length and width distributions, then when the model pulled its information from the Weibull distribution it would not have been as accurate.

In comparing the Monte Carlo model output to the previous work done by SKA, it can be seen that having a short period of observations creates serious problems. With only a relatively small number of years of data, it is hard to see if those years are typical tornado years, or if they are large or small tornado years. With millions of tornado events from the statistical distributions and Monte Carlo model, we can have more confidence in the maxima and minima that are produced; the big and small years do not stand out enough to create many small peaks in tornado probability across the United States. An important, albeit almost impossible to answer question, is how well the assumed distribtions fit reality. This is made more difficult than for some other problems of this nature by the changes that have occurred over time to the observations, such as the apparent increase in reported width of F4 tornadoes since 1981. We know that the reports do not correspond exactly to meteorological “truth”, but the relationship is almost impossible to determine and, unfortunately, it is likely to be different in different locations for a variety of reasons, such as population density (e.g., Doswell and Burgess 1988, King 1997).

In addition, the assumption that the most of the parameters are constant across the United States, there is the possibility that real, small-scale features in the climatology are being ignored. Since the statistical model looks over the entire United States and not over smaller sections at a time, there might be spatial variability that is missed. The model also holds everything constant (path length, path width, F-scale, number of tornadoes) across the entire United States except for the probability of a tornado occurring. So, given a location where a tornado is occurring, all of the other information about the event at hand is then drawn from general statistics. While this is reliable and does give a smooth graph, there are certain features specific to regions of the United States that might not be seen.

We intend to continue development of this, and similar models, in order to provide more reliable and consistent estimates of severe weather hazards in the United States. Brooks and Doswell (2001) suggest that at least some important features of severe weather climatology are consistent over a wide range of locations.To the extent that is true, it provides hope that statistically-based models of severe weather can be developed that esimate the threats in other countries as well. Using this class of model, in conjunction with studies of the climatology of the environmental conditions associated with severe weather seems to be an opportunity to determine what typical (and atypical) severe weather serasons look like.

In the future, the likelihood of a tornado occurring at any one point or on a specific day in the United States might be modeled, and the risks can then be evaluated for specific urban areas or areas of interest. The Monte Carlo model can be run for longer periods of time, and the variability between model runs can be analyzed. The more years of useful and reliable tornado output that are produced, the closer we come to having a definite hazard model for tornadoes. With a definite hazard model, risk management officials such as insurance companies and emergency response people will have the knowledge of when tornadoes are most likely to occur in their areas, and they will be able to be aware of the risk of a significant event on a specific day. The public will also be able to know the return probability of a significant tornado in their community, and people and businesses can prepare for such an event appropriately.

5. Acknowledgments

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