SIGNAL DESIGN AND PROCESSING TECHNIQUES FOR WSR-88D AMBIGUITY RESOLUTION

PART - 3

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1. Introduction.

The Operational Support Facility (OSF) of the National Weather Service (NWS) has funded the National Severe Storms Laboratory (NSSL), the National Center for Atmospheric Research (NCAR), and the Forecast Systems Laboratory (FSL) to address the mitigation of range and velocity ambiguities in the WSR-88D. This is the third report in the series that deals with velocity and range ambiguity resolution in the WSR-88D. The first two reports mainly dealt with uniform PRT transmission and phase coding techniques to resolve the range ambiguity. Although the phase coding techniques do not directly address the velocity ambiguity problem, its capability of separating overlaid echoes allows the use of shorter PRTs which, in turn, diminishes the occurrence of ambiguous velocities. In this third part, we consider the staggered PRT technique and its variants. Because the pulse repetition time is varied in all these schemes, we classify these methods as time coding methods. A comparison of the SZ phase coding technique and the time coding techniques is also carried out to arrive at a best strategy to be adopted for ambiguity resolution in the WSR-88D.

There are several variants of the time coding schemes reported in the literature. Notable among them are (i) staggered PRT technique (Zrnic and Mahapatra, 1985), (ii) spaced pair with polarization coding (Doviak and Sirmans, 1973), (iii) interlaced sampling (Sirmans et. al. 1976; Doviak and Zrnic, 1993), (iv) multi-PRI scheme suggested by Lincoln Lab, MIT, (Chornoboy and Weber, 1994), (v) alternating codes suggested by Finnish scientists (Pirttila et. al., 1999). A detailed discussion on some of the methods is available in the book by Doviak and Zrnic (1984). Conceptually, some of these methods can provide a large unambiguous velocity and range. However, their practical utility has been limited by three major problems: (a) the difficulty in filtering the ground clutter, (b) resolving overlaid echoes, and (c) obtaining spectral moment estimates with a reasonably low variance. The clutter filtering introduces unacceptably large bias error in velocity estimates in certain Doppler bands, and the overlaid echoes increase the variance of the spectral estimates.

Most promising among the schemes listed in the previous paragraph are the staggered PRT schemes, and they are possible candidates at least for the higher elevation scans in the WSR-88D. Therefore, in addition to examining all the available schemes, extra effort was put in solving the outstanding problems of the staggered PRT schemes; viz., the clutter filtering and improving the variance of the estimates. We shall focus on the staggered PRT scheme in which 2 PRTs are alternately transmitted.

At least half a dozen different ways of processing the staggered PRT samples were examined using simulation procedures, before a successful solution was arrived at. All these methods are not included in this report, but only the successful method has been given. Here we briefly recount the story of investigations that led to the successful solution to the clutter filtering problem. The decreased variance of the spectral moment estimates was an additional gain that results from the new procedure.

To begin with, we started from an examination of the available results in Zrnic and Mahapatra (1985) and Banjanin and Zrnic (1991). Banjanin and Zrnic (1991) have used a pair of filters on uniform sample sequences derived from the staggered PRT sequence. Torres (1998) has investigated the use of an orthogonal polynomial based regression filter on the non-uniform staggered PRT samples directly. These efforts have indicated that the rejection bands are not easily removed. This is because the problem is in the non-uniform sampling itself and not in the clutter filtering method. The non-uniform sampling produces aliasing of some of the signal power into the clutter spectral regions, and hence, these portions are indistinguishable from the clutter.

The rejection bands in the clutter filter can be traced to the periodicity in the staggered PRT sampling scheme. Therefore, our first approach was to remove the periodicity in the sampling wave form with the hope that this will remove the rejection bands in the clutter filter or smear the rejection bands over the entire spectrum while greatly reducing the attenuation. There are two ways this could be done: (i) vary the PRTs randomly about some mean value, or (ii) switch between the two PRTs, T_1 and T_2, randomly. In the first case, computation of autocorrelation is complicated; hence, the second method was studied using simulation. It was observed that with randomly switched PRTs, the variance of the velocity estimate is higher ($sd(v)$ was 2 to 3 times larger), and the clutter filter rejection bands do spread out and have a much lower attenuation. The rejection level could be reduced to almost 1dB and spread throughout the spectrum, but at the expense of an increased error in the velocity estimate.

In order to improve the error performance, next we considered spaced pair transmission. The pulse spacing T_1 is made small so that the velocity can be computed from $R(T_1)$ directly rather than using the difference, as in the staggered PRT scheme. Although it is well known that signal overlay cannot be avoided in this case unless polarization switching is used, we encounter other problems such as isolation between the two channels, differential propagation phase shift, etc., which need to be considered in evolving an algorithm for spectral moment estimation. Therefore, the spaced pair scheme was considered without the polarization switching, and with the hope that some kind of phase coding can be worked out to separate the overlaid signals once an effective way of clutter filtering is designed. The advantage seen here is the lower $sd(v)$ available with a spaced pair.

A successful clutter filtering procedure was developed for the spaced pairs scheme, which involved polynomial interpolation to reconstruct an equivalent uniform PRT sequence from the spaced pair sequence of complex samples. Since the clutter signal is of low frequency, the reconstruction is very good; therefore, a spectral domain filter can reduce the bias error in the velocity estimate to a reasonably low value while still maintaining a low $sd(v)$. However, we could not make much headway in separating the overlaid signal with phase coding, without which the spaced pairs scheme is not useful in practice.

The polynomial interpolation was then tried on the staggered PRT sample sequence, but with much less success. The clutter filtering was effective, but the rejection bands in the frequency response of the clutter filter, which produced large bias errors in the velocity estimate, were still present. In all the simulation studies of the clutter filtering, it was observed that the bias error is systematically negative for positive velocities and positive for negative velocities. It was logical to look for a clutter filtering scheme which produced an exactly opposite bias so that it can be canceled. The result of this search produced the new algorithm.

A major turning point in the search for a clutter filtering method occurred when we started looking at the staggered PRT signal in the spectral domain. This gave us an insight into the spectral properties of the staggered PRT signal which subsequently lead to the development of an effective clutter filtering scheme and also resulted in an improvement in the spectral moment estimates. Thus, two problems were solved at the same time. The new algorithm developed for the staggered PRT scheme overcomes the problem of clutter filtering completely. The algorithm also estimates the spectral moments with much lower variance than that reported earlier. With this new algorithm, it becomes practical to use the staggered PRT scheme to achieve a larger unambiguous velocity and range with acceptable error levels in the Doppler spectral moment estimates.

Although the study was started with an intention of an in depth examination of other available time coding schemes (e.g., the block staggered, spaced pair with polarization switching, etc.) to evaluate their suitability to the WSR-88D ambiguity problem, the discovery of the new clutter filtering and spectral processing method, whose performance surpassed all our expectations, has prompted a change in the emphasis so that a major part of the report is devoted to the delineation of the new algorithm for the staggered PRT scheme and its performance.

2. The staggered PRT technique.

Here, we describe the staggered PRT scheme briefly before we embark on a discussion of the new method of processing. In the staggered PRT technique (Zrnic and Mahapatra, 1985), two different pulse spacings, T_1 and T_2, are used alternately (Fig. 2.1a). Then, alternate pairs of return samples are used to compute autocorrelation estimates, R_1 at lag T_1 and R_2 at lag T_2. The velocity is estimated from the phase difference between the two using the formula,

$$\hat{v} = \lambda \, arg(R_1 R_2^*) \; / \; [4\pi(T_2 - T_1)] \; . \qquad (2.1)$$

Thus, the difference in PRT, $(T_2\text{-}T_1)$, determines the unambiguous velocity, v_a, for the staggered PRT technique and is given by

$$v_a = \pm \lambda \, / \, [4(T_2\text{-}T_1)] \ ; \quad T_1 < T_2 \, . \qquad (2.2)$$

Zrnic and Mahapatra (1985) suggest a testing procedure to estimate mean velocity and signal power for echoes received within the time delay (T_1+T_2). In theory, this seems to be possible because the overlaid signals in any two consecutive samples are from two different ranges and, therefore, are uncorrelated. Thus, the expected value of the overlaid signal contribution to the autocorrelation is zero, and the effective unambiguous range becomes

$$r_a = c(T_1+T_2)/2. \qquad (2.3)$$

Eq. 2.1 and 2.3 suggest that the staggered PRT is equivalent to a uniform PRT = (T_1+T_2) for the unambiguous range and a uniform PRT = $(T_2\text{-}T_1)$ for the unambiguous velocity, and each can be selected independently. However, the practical utility of this scheme is limited due to the quality of estimates. The overlaid signal increases the variance of the estimates because it acts as noise. Thus, the ratio of the overlaid signal powers is the equivalent signal-to-noise ratio (SNR), and for a reasonable accuracy of the estimates, the unwanted signal has to be at least 3 dB below the desired signal power.

Let $r_{a1} = cT_1/2$ and $r_{a2} = cT_2/2$ so that $r_a = r_{a1} + r_{a2}$. If r_{a1} is chosen sufficiently large so that no echoes are received from ranges greater than r_{a1}, then the problem of overlaid echoes could be eliminated. For weather radars, r_{a1} would have to be 460 km (for 0.5 deg. elevation scan), but this would degrade the variance of the estimates considerably. Thus, the practical limit for r_{a1} is smaller than 460 km unless some means of separating the overlaid signals is employed. It may be possible to extend the unambiguous range to r_{a2} with some additional processing to resolve the resulting single signal overlay (i.e., alternate samples only have overlaid echoes) in some of the range gates. This possibility will be explored in the future, but in this report, we consider data which have no overlay.

It is shown by Zrnic and Mahapatra (1985) that the standard error in the velocity estimate increases as the ratio $\kappa = T_1/T_2$ approaches unity, and a good choice is κ =2/3. Thus, the unambiguous range and unambiguous velocity are indirectly tied in practice via the estimate accuracy. However, compared to the uniform PRT, it is possible to achieve a much larger r_a and v_a because the limiting equation is $v_a\, r_{a1} = [\kappa/(1-\kappa)\}c\lambda/8$ for the staggered PRT scheme.

A major problem with staggered PRT scheme has been the ground clutter filtering. The non-uniform sampling aliases power from certain Doppler frequencies into the clutter frequency band around zero Doppler. Then, filtering the clutter also removes the aliased signal power from a band of spectral coefficients and introduces phase perturbations at these bands which bias the velocity estimate. The widths of these bands depend on the spectrum width of the signal as well as the clutter filter width. Banjanin and Zrnic (1991) have investigated several methods of ground clutter filtering to mitigate the phase perturbations. A scheme they proposed uses two filters sequentially such that the overall filter coefficients are time varying. In the Doppler bands where the filter phase response is not linear, special decision logic corrects velocity estimates. To overcome these obstacles, Chornoboy (1993) proposed a processing technique applied to a block staggered sampling and a least squares design of a filter matrix to achieve a desired frequency response. The added complexity of the pulse pattern enables an improved balance between the magnitude and the phase response so that Chornoboy(1993) achieved satisfactory results.

In the following sections, we present a different and novel approach to the clutter filtering and spectral moment estimation for the staggered PRT sequence.

3. A new approach to processing the staggered PRT samples:

In the proposed new approach, we seek to reconstruct the spectrum of the weather signal from the staggered time series samples, i.e., reconstruct the spectrum of the time series with a uniform sampling period of T_u, starting from the staggered PRT samples sequence, and then estimate the spectral moments from this reconstructed spectrum. This procedure allows estimation of the spectral moments with a much lower variance than the earlier methods. Further, a novel method of clutter filtering in the spectral domain has been proposed which can achieve a clutter suppression in excess of 40 dB and near complete elimination of all the spurious rejection bands in the $\pm v_a$ interval encountered by other methods of clutter filtering. This is the most important feature of the clutter suppression scheme which makes it practical to use the staggered PRT scheme in weather radars. The processing procedure has two major parts, 1) the reconstruction of an equivalent uniform PRT sequence for spectral moment estimation, and 2) the clutter filtering and residual bias removal. These two are interleaved because the clutter has to be filtered before the spectral moments are estimated. We will explain the reconstruction of the equivalent uniform PRT sequence first however, and then proceed to explain the clutter filtering procedure. These two procedures are incorporated in the algorithm presented later in this report.

3.1. Reconstruction of the signal spectrum.

Our technique requires a small restriction on the selection of the two PRTs used in the staggered PRT transmission. If T_1 and T_2 are the PRTs, we select them such that they are integer multiples of some basic PRT T_u, so that $T_1=n_1T_u$, and $T_2=n_2T_u$, where n_1 and n_2 are integers. Although n_1 and n_2 can be any integers in general, a good choice is $n_2=n_1+1$ from the point of the performance of the staggered PRT scheme. Thus, $(T_2-T_1)=(n_2-n_1)T_u$ determines the unambiguous velocity, v_a, and the unambiguous range, $r_{a1}=cT_1/2$. This, of course, assumes that T_1 is chosen sufficiently large so that no second trip overlay occurs.

Let g_i, i=1,2,3,...M, (M even) be the samples of the weather signal sampled at time intervals T_1 and T_2, alternately. We introduce zeros in g_i to form a sample sequence v_i of length $N=(n_1+n_2)M/2$ with a uniform sampling period of T_u, in which the missing samples are represented by zeros (see Fig. 2.1b). We call this the derived time series. Let c_i be a code sequence of length N obtained by replacing all the g_i samples in v_i by unity. For example, c_i =[1010010100... etc.] for $\kappa = T_1/T_2 = n_1/n_2 = 2/3$. We can write the sample sequence v_i as a product of the sequence c_i and e_i, where e_i is the signal time series sampled at T_u intervals.

$$v_i = c_i\, e_i\ ; \quad i=1,2,3,\dots N. \qquad (3.1)$$

Having converted the staggered PRT sequence into a uniformly sampled sequence (with missing samples represented by zeros), we can examine the spectrum of the uniform sequence, e_i. Thus, the spectrum of v_i can be represented as a convolution of the spectrum of the code c_i and the spectrum of the complete but unknown signal e_i.

$$\text{DFT}(\ v_i\ ) = \{\ \text{DFT}(\ c_i\ ) \star \text{DFT}(\ e_i\ )\ \} \tag{3.2}$$

where $\star$ represents circular convolution, and DFT( ) represents the discrete Fourier transform of the sequence in brackets. We use the capital letters to denote the spectral coefficients of the corresponding time domain quantities denoted by lowercase letters and capital bold face letters to denote matrices. Subscript index 'i' is used for the time domain quantities, and subscript index 'k' is used for the spectral coefficients. For example, $E_k = \text{DFT}(e_i)$, are the spectral coefficients, and **E** is the column matrix of coefficients E_k . Eq. (3.2) can be written in matrix form as

$$\mathbf{V} = \mathbf{C}\ \mathbf{E}. \tag{3.3}$$

V and **E** are (Nx1) column matrices containing the spectral coefficients, V_k and E_k, of the corresponding time sequences, v_i and e_i, and **C** is the convolution matrix (size: NxN) whose column vectors are cyclically shifted versions of C_k . To preserve the power in the spectrum, the convolution matrix, **C**, is normalized such that each column vector is a unit vector (i.e., the norm of each column vector is unity). Note that by normalizing the column vectors, the row vectors are also normalized automatically. Our objective here is to reconstruct the spectrum E_k from the samples g_i. It is observed that the convolution matrix is singular (rank of **C** equals the number of staggered PRT samples, M), and hence, it cannot be inverted to get E_k. If we discard the phases of the convolution matrix elements, the matrix becomes non-singular. It may be noted that the magnitude of the spectral coefficients $abs\{E_k\}$ is sufficient to compute the autocorrelation $R(T_u)$, and hence, the velocity and spectrum width. Therefore, we attempt to retrieve $abs(\mathbf{E})$ using $abs(\mathbf{C})$, which can be inverted.

In general, $abs\{\mathbf{CE}\} \neq abs\{\mathbf{C}\}abs\{\mathbf{E}\}$ because of the complex addition of the coefficients in the process of matrix multiplication. However, because the convolution matrix, **C**, has only (n_1+n_2) non-zero coefficients separated by $N/(n_1+n_2)$ coefficients for $\kappa = n_1/n_2$ in each row (or column), we can show that the complex addition does not take place in the process of convolution if the signal spectrum is narrow so that its spread is less than $N/(n_1+n_2)$ coefficients, and hence, the equality is valid. Then, the convolved matrix **V** has (n_1+n_2) spectral replicas of the original spectrum, **E** (in the complex domain, each replica has a specific complex multiplier), and these replicas do not overlap. Thus, we define a "magnitude deconvolution" as

$$abs\{\mathbf{E}\} = [\ abs\{\mathbf{C}\}]^{-1}\ abs\{\mathbf{V}\}, \qquad (3.4)$$

where $[\ abs\{\mathbf{C}\}]^{-1}$ is the magnitude deconvolution matrix. It is important to note that this equation provides the exact magnitude spectrum only under the condition that the non-zero spectral coefficients of the signal e_i spread at most $N/(n_1+n_2)$ coefficients or that the total spread must be less than

$$2v_a/(n_1+n_2) \qquad (3.5)$$

in the velocity domain. This condition means the spectrum cannot alias on itself and is obtained from the average sampling rate $2/(T_1+T_2)$ for the staggered sequence. Although wider spectra are not reproduced with high fidelity using this procedure (because of the overlap of spectral coefficients in the spectrum V_k), the non-ideal reconstruction of wider spectra does not generally bias the velocity estimate but affects its variance. If T_1 and T_2 are judiciously chosen, the criterion (Eq. 3.5) can be nearly satisfied for most weather signals. For example, at a 10 cm wavelength and $v_a = \pm 50$ m s^{-1}, Eq. 3.5 can be nearly satisfied for width, $w<6$ m s^{-1}. Nonetheless, it is important to note that the criteria is not satisfied exactly even for $w=4$ m s^{-1}.

Most of the results presented in this report are based on simulation studies. All the computations were carried out using simulated staggered PRT time series. The staggered PRT sequence is generated from a simulated uniform sample sequence by dropping the missing samples. The time series simulation procedure is explained in Part-1 of this report. There is an inherent rectangular window associated with a finite sample sequence in practice. This effect is simulated by generating a long time series and then truncating it. The effect of the rectangular window is to spread the power in the spectral domain, resulting in overestimation of the spectrum width. To obtain the true spectrum width, it is necessary to multiply the time sequence by a suitable tapered window function before processing. For Gaussian spectra the von Hann window function works very well.

Fig. 3.1 illustrates the spectrum reconstruction using a simulated weather signal time series. A 10 cm wavelength is assumed, and the magnitude spectrum of the signal time series (signal sampled at a uniform sampling period of T_u=0.5 ms) is shown in Fig. 3.1a. The normalized magnitude spectrum of the code for staggered PRT sampling with κ =2/3 (code is 1010010100...etc.) is shown in Fig. 3.1b. If the staggered PRT sampling were used (T_1=1 ms, T_2=1.5 ms), the spectrum of the derived uniform time series would be as given in Fig. 3.1c, which is the convolution of the first two spectra. The last spectrum (Fig. 3.1d) is the reconstructed spectrum from the staggered PRT samples using the magnitude domain deconvolution procedure. Note that for w=4 m s^{-1}, the spectrum does not exactly satisfy the criteria (3.5); hence, a few residual spectral coefficients remain throughout the spectrum. For larger width signals, the residuals can be significant because of the spectrum overlap, but this is not a cause for concern because the residuals maintain a symmetry about the mean Doppler, and hence, do not bias the mean velocity estimates. The residuals do bias the width estimates, and the spectrum needs to be modified a little by dropping some of the coefficients before the width is estimated. Specifically, for κ=2/3, dropping all the spectral coefficients outside a 2/5th of the spectral coefficients centered on the mean Doppler produces the best width estimates; a small bias remains at widths larger than 6 m s^{-1}. The spectrum width is computed using Eq. 6.37 of Doviak and Zrnic (1993). It is important to note that the velocity is estimated with all the coefficients before truncating the spectrum, and this estimated velocity is used for determining the extent of spectrum to be retained for width computation. The velocity computed from the truncated spectrum will have larger errors (variance as well as bias), and hence, only the width is computed after truncating the spectrum.

Once the magnitude spectrum is obtained, the spectral domain equivalent of the pulse pair algorithm can be used to estimate the mean power, p, mean velocity, v, and spectrum width, w. The autocorrelation, $R(T_u)$, can be expressed as

$$R(T_u) = 1/N \sum_{k=1}^{N} |E_k|^2 \, e^{j2\pi(k-1)/N}, \tag{3.6}$$

and the velocity is obtained from the phase of $R(T_u)$. The conventional pulse pair algorithm can be used for mean power and width estimation. Since the time series is obtained by inserting zeros in place of missing samples, a correction factor of $(n_1+n_2)/2$ has to be applied to get the correct mean power estimate. But it is best to obtain signal power from the original staggered PRT samples directly if the clutter filtering procedure (explained later in this report) is not applied. If the ground clutter is present in the time series, the signal power estimation must be done after the clutter filtering and bias removal procedures.

An important advantage of this procedure is that the estimate variances are much lower compared to the method using $R(T_1)$ and $R(T_2)$. Because the procedure is non-linear, it was deemed that the derivations of the theoretical expressions for the variance of the estimates would be too laborious, and the results would be only approximately valid. Therefore, we chose simulations to obtain the standard errors of the velocity estimate, $sd(v)$, and other statistical parameters. Results from one typical simulation run is presented in Fig. 3.2, which shows estimated velocity versus the input velocity. Each point on the figure represents one realization, and the continuous curve is the mean value. Figs. 3.3 and 3.4 show the bias in the estimated width using the simulation procedure without and with the von Hann window. The figures also show that the standard error in the width estimates is small. Each point is one realization, and the continuous curve shows the bias in the width. There is an overestimation of the width for narrower signals without the von Hann window. With the von Hann window weights applied, this bias is removed, and the estimates are better. However, for velocity estimation, it is not necessary to apply the window; in fact, the velocity estimates have lower variance without the window than with the window applied because some information is lost. If one estimated the spectrum width without using the von Hann window, but accounted for the bias by some other means, the variance of the width estimate also would be lower. If clutter is present and is to be filtered, then application of the von Hann window weights becomes necessary.

The standard errors of the spectral moments obtained from the conventional method and the present method (using the magnitude deconvolution) are compared in the next two figures. In Fig. 3.5, the variation of *sd(v)* as a function of the spectrum width is plotted for (a) the pulse pair velocity estimator that uses the complete time series with a sampling period T_u and length N, (b) the proposed method using the reconstructed spectrum from staggered PRT samples, and (c) the pulse pair velocity estimator that uses the phase of $[R(T_1)/R(T_2)]$. For each spectrum width, a large number of simulations were carried out to obtain the value of the *sd(v)*. There is a significant improvement in the *sd(v)* using the present method, and it compares very well with that of the pulse pair estimator on a complete time series for widths up to 5 m s^{-1}. The spectrum reconstruction is good for widths up to 6 m s^{-1}.

Because the spectrum is reconstructed, it is possible to devise a width estimator which has smaller standard error than what ensues from $log[|R(T_1)/R(T_2)]$. For example, the estimator based on $log[|R(0)/R(T_u)]$ (Eq. 6.37 in Doviak and Zrnic, 1993) yields at least a three fold decrease in the standard error of the estimate - almost the same as the pulse pair estimator using the complete time series. Since the spectrum is reconstructed, reflectivity and width also have similar standard error behavior with respect to the width of the signal. Fig. 3.6 shows the *sd*(*w*) computed from extensive simulations. The standard error in the spectrum width estimate also closely follows that of the equivalent uniform PRT.

3.2. Ground clutter filtering.

The clutter filtering is carried out in the spectral domain before the signal spectrum is reconstructed using the magnitude deconvolution procedure described in Section 3(a). The spectrum of the ground clutter returns is assumed to be of narrow width and centered on the zero Doppler. Because the spectrum of the derived time series, v_i, is the convolution of the code spectrum and the signal spectrum, it will have weighted replicas of the clutter spectrum centered on each of the non-zero spectral lines of the code spectrum. The weights are the spectral coefficients of the code spectrum. An example of the convolved spectrum is shown in Fig. 3.7c for $\kappa = T_1/T_2 = 2/3$. The method to be described is not confined to κ=2/3, but this example is used only to illustrate the clutter filtering procedure. Because the code spectrum in this case has five non-zero spectral coefficients (see Fig. 3.1b), the power from each of the clutter spectral coefficients is spread over five spectral coefficients separated by $N/5$ coefficients in the convolved spectrum. Specifically, power in the k^{th} spectral coefficient is distributed over spectral coefficients at k, $k+M/2$, $k+M$, $k+3M/2$, and $k+2M$, cyclically (i.e., if any of these indices exceed N, subtract N from the number). This type of modulation also affects the weather signal so that its spectrum, too, would be replicated into five bands. The modulation could cause weather and clutter that did not overlap in the unmodulated spectrum to overlap after modulation. In general, the power in each spectral coefficient is spread into only (n_1+n_2) coefficients separated by $N/(n_1+n_2)$ coefficients. Therefore, we can take (n_1+n_2) coefficients at a time and try to restore the spectrum. This is done by rearranging the matrices as described below.

Consider an example with parameters κ=2/3, M=64 for which N=160. Because the convolution matrix has only five non-zero coefficients in each column (or row), we can recast the matrix Eq. 3.3 in the form

$$\mathbf{V}_r = \mathbf{C}_r\, \mathbf{E}_r, \tag{3.7}$$

where subscript 'r' is used to represent rearranged matrices, which are given by

$$V_r = \begin{bmatrix} V_1 & V_2 & V_3 \cdots V_{32} \\ V_{33} & V_{34} & V_{35} \cdots V_{64} \\ V_{65} & V_{66} & V_{67} \cdots V_{96} \\ V_{97} & V_{98} & V_{99} \cdots V_{128} \\ V_{129} & V_{130} & V_{131} \cdots V_{160} \end{bmatrix}, \tag{3.8}$$

$$C_r = \begin{bmatrix} C_1 & C_{129} & C_{97} & C_{65} & C_{33} \\ C_{33} & C_1 & C_{129} & C_{97} & C_{65} \\ C_{65} & C_{33} & C_1 & C_{129} & C_{97} \\ C_{97} & C_{65} & C_{33} & C_1 & C_{129} \\ C_{129} & C_{97} & C_{65} & C_{33} & C_1 \end{bmatrix}, \tag{3.9}$$

and

$$E_r = \begin{bmatrix} E_1 & E_2 & E_3 \ldots E_{32} \\ E_{33} & E_{34} & E_{35} \ldots E_{64} \\ E_{65} & E_{66} & E_{67} \ldots E_{96} \\ E_{97} & E_{98} & E_{99} \ldots E_{128} \\ E_{129} & E_{130} & E_{131} \ldots E_{160} \end{bmatrix}. \tag{3.10}$$

The convolution matrix, **C**, is modified by deleting first all the rows containing zero in the first column and then deleting all columns containing zero in the first row, which reduces it to a 5x5 matrix, $\mathbf{C}_r$, and the other two matrices are rearranged as 5x32 matrices, row wise as in Eq. 3.8 and Eq. 3.10. The Matrix $\mathbf{C}_r$ is singular (its rank is 3), and hence, cannot be inverted; its columns are normalized such that the sum of the magnitudes squared of the elements in each column is unity. (This also normalizes the row vectors.) In the complete time series spectrum, E_k, the clutter is confined to the first and last few coefficients (k=1,2,..q and N-q-2, ... N). But the matrix $\mathbf{V}_r$ would have clutter power spread over the first few and the last few columns.

To filter the clutter from the first few columns of $\mathbf{V}_r$, first estimate the complex amplitude of the clutter vector in the column (which also might contain a signal component). This is accomplished by taking the inner product of the column with the complex conjugate of the first column vector of the matrix $\mathbf{C}_r$. The clutter column vector is constructed by multiplying the first column vector of $\mathbf{C}_r$ by this complex amplitude coefficient, and is then subtracted from the corresponding column vector of matrix $\mathbf{V}_r$. This process is carried out for the first few columns containing clutter. Similarly, the clutter from the last few columns is removed using the last column vector of $\mathbf{C}_r$ in place of the first. This complete operation can be written in matrix notation as

$$\mathbf{V}_f = \mathbf{V}_r - \mathbf{C}_{f1}\,\mathbf{V}_r\,\mathbf{I}_{f1} - \mathbf{C}_{f2}\,\mathbf{V}_r\,\mathbf{I}_{f2}, \qquad (3.11)$$

where $\mathbf{C}_{f1}$ and $\mathbf{C}_{f2}$ are the clutter filter matrices, and $\mathbf{I}_{f1}$ and $\mathbf{I}_{f2}$ are matrices which select the columns to be filtered. These are given by

$$\mathbf{C}_{f1} = \mathbf{C}_1\,\mathbf{C}_1^{t*}, \qquad (3.12a)$$

and

$$\mathbf{C}_{f2} = \mathbf{C}_5\,\mathbf{C}_5^{t*}, \qquad (3.12b)$$

where $\mathbf{C}_1$ is the first column vector of $\mathbf{C}_r$, $\mathbf{C}_5$ is the last column vector (5^{th} column for κ=2/3) of $\mathbf{C}_r$, the symbol * indicates complex conjugate, and the superscript "t" stands for transpose. The matrix $\mathbf{I}_{f1}$ is a $M/2$x$M/2$ diagonal matrix with diagonal elements unity for the first q elements and the rest zeros. Similarly, the matrix $\mathbf{I}_{f2}$ is a $M/2$x$M/2$ diagonal matrix with last (q-1) elements unity and the rest zeros. For example, the diagonals of the matrices for q=4 are

$$diag\ \{\mathbf{I}_{f1}\} = [11110000\ ....\ 000],$$

and

$$diag\ \{\mathbf{I}_{f2}\} = [0000\ ....\ 000111]. \qquad (3.13)$$

These matrices determine the clutter filter width. Note that $(2q\text{-}1) = n_c$ is the number of columns containing the clutter we wish to filter, or it is the clutter filter width in terms of the number of spectral coefficients. This number should be determined from the expected width of the clutter signal. The matrix operation indicated in Eq. 3.11 produces the filtered signal matrix $\mathbf{V}_f$.

Now, to reconstruct the magnitude spectrum of the weather signal, apply the magnitude deconvolution procedure to $\mathbf{V}_f$, as explained earlier (see Eq. 3.4). This can be done in the rearranged matrix form by inverting Eq. 3.7, and is given by

$$\mathbf{E}_r = [abs\{\mathbf{C}_r\}]^{-1}\, abs\{\mathbf{V}_f\}. \qquad (3.14)$$

This is faster than using Eq. 3.4 because it involves inverting a 5x5 matrix in place of the NxN matrix. The magnitude spectrum $abs\{\mathbf{E}_r\}$ is rearranged in a column matrix $\mathbf{E}_s$. The autocorrelation $R(T_u)$ is calculated from the magnitude coefficients of $\mathbf{E}_s$ using Eq. 3.6, and the velocity estimate from the phase of $R(T_u)$.

In Fig. 3.7 is an illustration of the clutter filtering procedure using a simulated signal and for the clutter-to-signal power ratio of 40dB. The spectrum of the complete signal (Fig. 3.7a) sampled with a period of $(T_2\text{-}T_1)$ is what we want to reconstruct from the staggered PRT samples. The clutter spectrum alone is in Fig. 3.7b. The spectrum of the signal plus clutter computed from the staggered PRT samples (or the derived uniform sequence) is in Fig. 3.7c. Note that the clutter is spread over five bins across the spectrum. The signal is not visible because it is 40 dB below the clutter. The spectrum after the clutter is filtered is given in Fig. 3.7d. The signal spectrum reconstructed by the magnitude deconvolution (Fig. 3.7e) has some residuals at the locations where clutter was present. Note that the residual coefficients at the places where clutter was removed all have the same amplitude; viz., coefficients # 1, 33, 65, 97, and 129 have the same amplitude; coefficients # 2, 34, 66, 98, and 130 have the same amplitude, etc. This fact and its relation to the actual signal amplitude is used later to remove the velocity bias. The spectrum after the residuals are deleted (Fig. 3.7f) is nearly the same as the original spectrum except for a few coefficients at the tail ends, and the coefficients from which clutter is filtered will also have an amplitude error which causes the bias.

Because the columns of $\mathbf{C}_r$ are not orthogonal, clutter filtering also filters some part of the signal power from the corresponding spectral coefficients. The signal component removed is the projection of the signal vector onto the clutter vector, and this results in an alteration of the relative amplitudes of the signal vector elements. Therefore, the vector does not deconvolve back into a single signal coefficient in the magnitude deconvolution process but produces residual spectral coefficients at the locations where clutter was present. However, the amplitudes of these residuals are comparable or less than the signal coefficients. The residuals do produce a small bias in the mean velocity estimate which is not appreciable for small clutter filter widths. The residuals are zero if the signal spectrum and the clutter spectrum do not overlap in the spectrum of staggered PRT sequence; if there is an overlap, the residuals can be significant compared to the signal coefficients but are nearly symmetrically placed about the mean velocity, hence produce a small bias. The maximum bias produced by the small asymmetry is about ±2 m s^{-1} for a clutter filter width of 4.4 m s^{-1}, $\kappa = 2/3$, and $v_a = 50$ m s^{-1}. For larger filter width, the bias increases proportionately, and systematically cycles between positive and negative maximum values as a function of the mean velocity. There are no significant dropouts in the velocity recovery. This very important feature of this clutter filtering procedure implies that the clutter filter response has no spurious rejection bands. The resulting velocity estimates might be good for some applications, but even this small bias can be removed by some additional processing as explained later in this section. The spectrum width estimate is heavily biased because of the residuals. A major part of the spectrum width bias is removed by deleting all the spectral coefficients outside $\{2N/(n_1+n_2)\}$ coefficients centered on the mean velocity, and then computing the spectrum width. It is the part of the bias produced by the residual spectral coefficients due to the spectrum overlap in the staggered PRT signal spectrum, as explained in the previous section (Section 3.a). There is also an additional bias produced because of the clutter filtering. This bias is comparable to that in the velocity and is a function of the mean velocity. This part of the width bias is eliminated in the process of removing the bias error in the velocity. The clutter filtering procedure also produces a bias in the signal power estimate, which is a function of the mean velocity as in the case of spectrum width bias. This bias also is removed in the bias removal procedure. Thus, the bias removal procedure removes the bias in all three spectral moment estimates.

This method was tested (without the bias correction part) using simulated staggered PRT weather signals with an unambiguous velocity interval of ±50 m s^{-1} and κ=2/3. Fig. 3.8 shows the mean estimated velocity as a function of the input velocity for one specific set of parameters indicated in the figure. All the published methods of clutter filtering produce clutter filter notches at velocities ±20 m s^{-1} and ±40 m s^{-1} (locations corresponding to the first and second alias of the sampling frequency $(T_1+T_2)^{-1}$) in addition to the notch at zero Doppler. Note that the velocity recovery is accurate at these values in the present method. There is a small bias between two of these notch locations because of the residual coefficients at spectral locations from which the clutter was removed, but it is within about ±2 m s^{-1}, and hence, would be acceptable for some applications. The bias error is maximum for mean velocities ±5, ±15, ±25, ±35, and ±45 m s^{-1}, and it is zero for ±10, ±20, ±30, ±40, and ±50 m s^{-1}. In Fig. 3.9, the bias and the standard deviation of the velocity estimate is shown for the same set of parameters in an expanded scale. There is a small increase in the standard error whenever the signal spectrum and the clutter spectrum overlap (i.e., at velocities 0, ±20, and ±40 m s^{-1}). For all these simulations, the clutter filter width used is $2v_a(9/160)$ (i.e., 9 coefficients out of 160 are removed). This is the filter width required to effectively remove clutter for a CSR =30 dB. For larger CSR values, clutter filter width also needs to be increased, and this produces larger bias error in the velocity estimate. Fig. 3.10 shows normalized maximum bias error in the velocity as a function of normalized clutter filter width, $(w_f/2v_a)$, obtained using a large number of simulations. The clutter filter width required for effective removal of the clutter, as judged by the velocity estimate quality, is given in Fig. 3.11. ($\zeta=w_f/w_c$; w_f is the clutter filter width, and w_c is the clutter spectrum width.) From these two figures, it can be inferred that for a clutter spectrum width of 0.35 m s^{-1} if the CSR is less than about 20 dB, the bias error in the velocity is less than ±1.5 m s^{-1} and might be tolerated. For larger CSR, we need to remove the velocity bias. The curve in Fig. 3.11 indicates that there is an upper limit for the signal velocity recovery in the presence of strong clutter. The CSR seems to level off around 50 dB with increasing width of the clutter filter, and about CSR=45 to 50 dB is about the best that can be filtered. At these highest CSR values, the velocity estimate has large standard error, especially if the signal and clutter overlap. For larger width signals (w >6 m s^{-1}), the reconstructed spectra are not accurate because of the spectrum overlap; hence, there is an increase in the variance of the velocity estimate.

Fig. 3.12 shows the clutter suppression ratio, $\alpha = -10 \log\{P_{res} / P_{tot}\}$, as a function of the normalized clutter filter width, ζ , ($\zeta = w_f / w_c$; w_f is the clutter filter width, and w_c is the clutter spectrum width) from simulation. The continuous curve is the theoretical curve for a Gaussian signal. Assuming that the clutter suppression ratio required is equal to the *CSR* plus the *SNR* (about 20 dB), we would expect velocity recovery for *CSR*=50dB for a clutter filter width, ζ=20 (see Fig. 3.12). But Fig. 3.11 indicates a velocity recovery only up to about *CSR*=40 dB for this clutter filter width. In fact, the velocity recovery is good up to *CSR*=50dB for regions where the clutter and signal do not overlap, but in the regions of overlap, there is an increase in the $sd(v)$ which sets an upper limit of *CSR*=40 dB.

The clutter filtering produces a bias in the width estimate as shown in one sample plot in Fig. 3.13 for an input width of 4 m s^{-1}. The points show each realization, and the mean value from several simulations is the continuous curve. It is clearly seen that the bias is a function of the mean velocity. This pattern continues for any input width. The amplitude of the bias in the width estimate is a function of the clutter filter width. This bias also is removed in the process of removing the bias error in the velocity discussed later in this section. The figure also shows the standard error in the width estimate which is less than 1 m s^{-1}.

3.3. Bias removal procedure.

It is mentioned in the previous paragraph that the clutter filtering from a column of the $\mathbf{V_r}$ alters the relative amplitudes of the signal components present in the column, thus preventing it from deconvolving into a single signal component in $\mathbf{V_f}$. A very interesting feature of $\mathbf{V_f}$ is that the amplitude of all the elements of the column vector from which clutter is filtered is modified such that after the magnitude deconvolution, the amplitude of all elements of the column vector is the same. Further, this amplitude bears a fixed amplitude relationship with the actual signal component present in the vector. This amplitude relationship is a function of the correct signal vector. There are 5 signal vectors for κ=2/3 (5 columns of the matrix $\mathbf{C_r}$

). The correct signal vector can be obtained from the estimated mean velocity. The five velocity regions which correspond to the five signal vectors are ($-v_a$ to $-3v_a/5$), ($-3v_a/5$ to $-v_a/5$), ($-v_a/5$ to $v_a/5$), ($v_a/5$ to $3v_a/5$), and ($3v_a/5$ to v_a). If we discard the sign of the spectral components, positive and negative velocities produce the same amplitude relationship. This relationship can be obtained by performing the matrix operations corresponding to the clutter filtering and magnitude deconvolution on each of the unit column vectors, $\mathbf{C}_1$ to $\mathbf{C}_5$, and is given by

$$\xi_k = 1 \;/\; abs\{\; [abs(\mathbf{C}_r)]^{-1}\; [\mathbf{C}_k - (\mathbf{C}_1^{t}{*}\mathbf{C}_k)\mathbf{C}_1]\}; \quad k=1,2,..(n_1+n_2), \qquad (3.15)$$

where $\mathbf{C}_r$ is the rearranged code matrix defined earlier, $\mathbf{C}_k$ is the k^{th} column vector of $\mathbf{C}_r$, and ξ_k is the ratio of the actual signal amplitude to the amplitude after magnitude deconvolution for the k^{th} velocity region. The ξ_k values are symmetric with respect to the zero velocity; i.e., the positive and negative velocities have the same value (except for the sign which is discarded because we are working in the magnitude domain). Therefore, only the first three values are taken for $\kappa=2/3$. Note that $\xi_1=\infty$, and hence, cannot be used for signal amplitude correction in the region $|v|<v_a/5$.

The ratio of the actual signal amplitude to the residual amplitude, ξ_k, obtained from simulation, is shown in Fig. 3.14 as a function of the input velocity. This is possible because the original signal is known for the simulation. In the staggered PRT processing algorithm, the estimated velocity is used in place of input velocity (Input velocity is available only for simulation!) to select the constant. The two constants for $\kappa=2/3$ are $\xi_2=1.1056$ and $\xi_3=1.789$ for regions $v_a/5<|v|< 3v_a/5$ and $|v|> 3v_a/5$, respectively. In the region around zero velocity, ($-v_a/5$ to $v_a/5$), the residual component is nearly zero, and hence, the ratio is very large. In and around the transition regions at $|v|= v_a/5$ and $3v_a/5$ (10 and 30 m s^{-1}), there is a rapid change in the constants, but this is of no concern because at these locations, the bias is already zero. (The residual spectral coefficients are symmetrically placed with respect to the mean velocity.) These fuzzy regions widen with increased spectrum width. These ξ values are independent of the number of samples, M, and also the selection of unambiguous velocity, v_a, for a given κ. The three regions of $\pm v_a$ are indicated in Fig. 3.14. For a different κ value, the number of regions as well as the ξ values for each region are different. (ξ values are given later in the report for other κ.)

The bias removal procedure is to reconstruct the original signal column vector whenever clutter is removed from a column vector of $\mathbf{V_r}$. To do this, first we calculate the mean velocity estimate from the spectrum $\mathbf{E_s}$ and determine the velocity region, (1), (2), or (3), to choose the multiplication coefficient, ξ. To explain the bias removal procedure, let us rearrange $\mathbf{E_s}$ into a (n_1+n_2)x$M/2$ matrix (5x32 for the present example), $\mathbf{E_{sr}}$, in which the first q and the last q-1 columns vectors, from which the clutter is filtered, will have same element values (different values for each column). Set all the elements of these columns to zero except one row. This row is determined by selecting elements in the $N/(n_1+n_2)$ spectral coefficients centered on the mean velocity estimate. Whichever row falls within this interval, the elements of that row of columns 1 to q are **not** set to zero but are multiplied by the constant ξ. Similarly, whichever row falls within this interval for the last (q-1) columns, the elements of that row of the last (q-1) columns are **not** set to zero but are multiplied by the constant ξ. This procedure is applied only if the estimated velocity is in regions (2) and (3), with multiplier ξ_k selected appropriately for the region (see Fig. 3.14 for regions). For region (1), the elements of this particular row, selected as in the previous case, are replaced by the $(q+1)^{th}$ element value for columns 1 to q, and the $(M/2-q+1)^{th}$ value for columns ($M/2$-q+2) to $M/2$. This, of course, is an approximation for want of an accurate ξ value. For this region, the clutter and signal vectors are the same, and hence, the signal is completely removed while removing the clutter. Therefore, we take the signal amplitude from the adjacent coefficient from which clutter is not filtered (e.g., $(q+1)^{th}$ column) and insert this value as an approximate signal amplitude. This, of course, is not the best choice but is taken as the simplest option. The bias removal is not exact for region (1) but is reasonably good for most applications. This corrected $\mathbf{E_{sr}}$ is again rearranged into a single column vector, and $R(T_u)$ and velocity are re-estimated from this column vector. This velocity is without any bias in the $\pm v_a$ interval. Thus, the additional computation to remove the bias error involves some matrix element manipulation and computation of autocorrelation and velocity a second time. The procedure is also applicable to other stagger ratios, κ.

This whole process of bias correction can be put in mathematical form using matrix