SIGNAL DESIGN AND PROCESSING TECHNIQUES FOR WSR-88D AMBIGUITY RESOLUTION

PART 2

National Severe Storms Laboratory Report

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Contents

LIST OF SYMBOLS:

c	-	speed of light
C_k	-	modulation code [$C_k = \exp(j\phi_k)$]
e_1, e_2	-	complex time series of 1st and 2nd trips
E_1	-	complex time series with 1st trip coherent and 2nd trip coded
E_2	-	complex time series with 2nd trip coherent and 1st trip coded
E_i	-	complex time series samples
$err()$	-	error in the parameter in brackets
i,k,n,m	-	used as indices
j	-	$(-1)^{1/2}$
M	-	number of samples
n_w	-	normalized notch filter width (normalized to $2v_a$)
p_1, p_2	-	mean powers of the 1st and 2nd trip signals
p_n	-	noise power
p_c	-	clutter power
P_k	-	power spectral coefficients
p_{coh}	-	coherent part of the power
p_{inc}	-	incoherent part of the power
r_a	-	unambiguous range
$r(k)$	-	random number array of length k
$R(n)$	-	autocorrelation for n pulse lag
R_c	-	clutter rejection ratio
R_p	-	residual power ratio (ratio of the power, p_1, to the residual $(p_1)_r$ after notch filtering)
R_{pe}	-	effective residual power ratio with random phase error in the phase shifter
R_{pt}	-	residual power ratio (ratio of the power (p_1+p_c), to the residual power of (p_1+p_c) after notch filtering)
R_o	-	overlapped power to total power ratio
s_k	-	k^{th} complex spectral coefficient
$sd()$	-	standard deviation of the parameter in brackets
S_1, S_2	-	spectrum of E_1 and E_2 [S_1= DFT(E_1)]
T	-	pulse repetition time
v_a	-	unambiguous velocity
v_m	-	mean velocity
v_1, v_2	-	mean velocities of the 1st and 2nd trip signals
w_1, w_2	-	spectrum widths of the 1st and 2nd trip signals
w_c	-	clutter spectrum width
w_{cf}	-	clutter filter width
z	-	$\exp(j2\pi/M)$
^	-	estimate

$\mathcal{P}$	-	probability
$\mathscr{E}$	-	expected value
τ	-	range time
ψ_k	-	switching phase code sequence
ϕ_k	-	modulation phase code sequence
μ	-	ratio of residual powers before and after subtraction

ABBREVIATIONS:

CD	-	Contiguous Doppler
CNR	-	Clutter-to-Noise Ratio
CS	-	Contiguous Surveillance
DFT	-	Discrete Fourier Transform
FFT	-	Fast Fourier Transform
GCF	-	Ground Clutter Filter
IDFT	-	Inverse Discrete Fourier Transform
PNF	-	Process Notch Filter (notch filter in the SZ decoding algorithm)
PRT	-	Pulse Repetition Time
SCR_1	-	Signal-to-Clutter ratio (1st trip signal)
SCR_2	-	Signal-to-Clutter ratio (2nd trip signal)
SNR	-	Signal-to-Noise Ratio
SNR_1	-	SNR of the 1st trip signal ($=p_1/p_n$)
SNR_2	-	SNR of the 2nd trip signal ($=p_2/p_n$)
$\pi/4$ code	-	$\{ 0, \pi/4, 0, \pi/4, ... \}$
$\pi/2$ code	-	$\{ 0, 0, \pi/2, \pi/2, ... \}$

SIGNAL DESIGN AND PROCESSING TECHNIQUES FOR WSR-88D AMBIGUITY RESOLUTION

PART 2

1. INTRODUCTION

The Operational Support Facility (OSF) of the National Weather Service (NWS) has funded the National Severe Storms Laboratory (NSSL), the National Center for Atmospheric Research (NCAR), and the Forecast Systems Laboratory (FSL) to address the mitigation of the range and velocity ambiguities in the WSR-88D system. This is Part 2 of the report on the ambiguity resolution. It documents the work done at the NSSL which was completed in the second year of the project.

In the first part of this report, several ambiguity resolution algorithms were studied theoretically, and extensive simulations were carried out to evaluate their performance. A comparison of the capabilities of each of these algorithms led to the selection of a systematic phase coding technique (the SZ code) as the best among the methods considered. In this second part of the report, we examine the properties of the SZ code in more detail, especially related to the processing steps used in the decoding algorithm. Several important practical aspects which were not included in the earlier simulation, and some aspects of implementation of the algorithm on the WSR-88D, are also included.

Some of the effects present in a practical radar signal are the window effect, ground clutter, and receiver noise. These are addressed here, and their contribution to the degradation in the performance of the systematic phase coding scheme is evaluated. A new and important consideration that needs scrutiny is the accuracy of the phase shifts that has to be maintained for the effective operation of the phase coding scheme. An analysis of the effect of random errors in the phase shifter on the performance of the algorithm, especially the velocity recovery, is carried out, and the results are presented. These practical aspects of the radar signal affect the performance of the different algorithms with respect to the recovery of the weaker signal velocity. Therefore, although the SZ(8/64) code was selected to be the best code in the first part of this report, we also study SZ codes with alternative values of (*n/M*), and with appropriately modified decoding algorithms, in an effort to evolve a phase coding scheme which performs best with all the practical constraints in place.

Another important practical consideration is the compatibility of the phase coding scheme with the present scan strategy of the WSR-88D. There are several parameters, such as the scan rate, the PRT, the number of samples, etc., that are pre-set in the WSR-88D, depending on the operating mode. The points to be addressed are: (a) how many of these parameters need to be changed in order to implement the phase coding scheme, and (b) how to integrate the phase coding scheme in the WSR-88D so that the meteorological performance of the radar is not compromised. Ideally, one would like to make the least amount of changes. However, if a change leads to a significant advantage without compromising existing capabilities, it should be made. All these, and many finer points of practical implementation of the algorithm, are discussed in this report.

With the implementation of the coding scheme to recover velocities of the first two trip echoes, we would have a range coverage of about 230 km for v_a= 32 m s^{-1} with a transmitting frequency of 3 GHz. The requirement of the reflectivity data over a range of 460 km at low elevation angles makes it imperative that we retain the present long PRT scan (Contiguous Surveillance mode of the WSR-88D). Some of the information obtained from this long PRT scan can be used in the short PRT Doppler scan (Contiguous Doppler mode) data processing to improve, speed up, and channel the computations along different paths in the algorithm.

At the intermediate elevation angles (2.5° to 6.5°), the batch mode of data acquisition is used, in which alternate radials have long and short PRT transmissions. Here also, the information obtained from the long PRT data can be used in the processing of the short PRT data. However, a change in the scanning mode of the WSR-88D is possible for these elevation angles because the lower edge of the beam at 2.5° elevation is above 11 km at ranges beyond 230 km. Thus, practically all of the storms detected at elevation angles 2.5° and above will be within 230 km, the range to which Doppler velocity processing is required. But we show that the SZ phase coding scheme can recover all the three parameters (i.e., if p_1/p_2<40dB) over twice the unambiguous range interval without the need for long PRT data. Thus, above 2.5° elevation, we can increase the data acquisition rate which can be used to decrease the scan time. Because of this possibility, we have developed the decoding algorithm in two different forms (SZ-1 and SZ-2). The first one works in a stand alone mode to recover all three spectral parameters of both trips, and the second algorithm recovers only the velocities of both trips and uses the long PRT scan data for the recovery of the reflectivity and the spectrum width. (The long PRT data in the present WSR-88D is used to estimate reflectivity only.) The second algorithm can be used if the long PRT data is available, and the first algorithm can be used if the batch mode is replaced by a phase coded Doppler scan.

Throughout this report, we use the following assumptions and notations for convenience: it is assumed that only the 1st and 2nd trip echoes are present in the radar signal, and the first trip is always stronger. The signal parameters, viz., the mean power, the mean velocity, and the spectrum width, are represented by p_i, v_i, and w_i, respectively, with the subscript i=1,2 representing 1st or the 2nd trip. Generally, the stronger signal parameters are easily recovered with the phase coding scheme; the limitation is in the recovery of the velocity of the weaker signal. Therefore, most of the discussions will be about the recovery of the velocity, v_2, of the weaker signal. When the spectral parameters are estimates, obtained from the time series (or spectrum), the symbol ^ is used to represent estimates. However, for convenience, this symbol has been omitted in many places. But it is clear from the context whether the parameter is an estimate or not.

In the lowest two elevation scans there is a possibility of multiple trip echoes. However, the SZ algorithm is developed for recovery of the first two trip signal parameters in the absence of the 3rd and 4th trip echoes. The multiple trip overlay case is discussed in some detail in section 5.6.

In Part 1 of this report, the error in the spectral parameters, estimated using the SZ decoding algorithm, were computed with respect to the autocovariance estimates of the same parameters obtained from the individual time series before they are combined to form the overlaid signal time series. This was done specifically to present the performance of the algorithms relative to the autocovariance processor, so that the comparison among the algorithms is made easier. This can give a false impression that the estimation error is zero in some cases (e.g., $err(\hat{v}_1)$ is zero for large p_1/p_2). Results presented in this report use errors computed with respect to the nominal parameters of the simulation, so that they represent realistic errors.

In Section 2, a comprehensive discussion on the SZ phase code, vis-a-vis the processing steps in the decoding algorithm, is presented. Section 3 is a detailed study of the various effects which are normally present in weather signals. Specifically, we address the effects of various window functions, white noise, and ground clutter filtering. Several practical aspects, which are specific to the phase coding scheme and the WSR-88D, such as the errors in the phase shifter and its effect on the performance of the phase coding scheme, the sample length selection, the code synchronization, etc., are discussed in Section 4. The two versions of the SZ decoding algorithms are given in Section 5 along with some results on the overall performance of the SZ phase coding schemes. Specifically, the performance of three SZ codes, viz., the SZ(8/64), SZ(12/64), and the SZ(16/64), is discussed and compared to arrive at an optimum code with all the practical effects included. A proposed schematic of the algorithm for implementation on the radar is also discussed in this section.

Whereas the simulated time series is a very good tool in the design stage of the phase coding scheme, it cannot represent all the variations in the actual radar derived time series. Due to the diversity in the weather phenomenon, there are situations which produce a non-Gaussian shaped spectra. Therefore, the actual performance of the phase coding scheme has to be obtained by testing it on real weather signals. This will be investigated in Part 3 of the report, which will be devoted to the study of the WSR-88D data. Here, simulated data fields are generated with overlaid echoes to demonstrate the performance of the algorithm. These results are discussed in Section 6 of this report. The conclusions drawn from this study are in the last section.

2. SZ PHASE CODING SCHEME

In a phase coded radar, the transmitter pulses are phase shifted by a pre-determined phase sequence, ψ_k, and the received echo samples are phase corrected (multiplied by $\exp\{-j\psi_k\}$) so that the 1st trip signal is coherent. However, the 2nd trip echo would not be cohered but will be modulated by a phase sequence $\phi_k = (\psi_{k-1} - \psi_k)$. If the second trip is cohered, then the 1st trip echo is modulated by a phase sequence $-\phi_k$. Here, ψ_k is the **SZ switching code** (phase shifter switching sequence), and ϕ_k is the **SZ modulation code.** In autocovariance processing, the mean velocity is estimated from the phase of the autocorrelation for lag 1, $R(1)$. The modulation code, ϕ_k ($-\phi_k$), modifies the spectrum of the 2nd (1st) trip echo so that its $R(1)$ is made zero; thus, the bias error in the velocity estimate of the cohered 1st (2nd) trip signal, due to the overlaid 2nd (1st) trip signal, is removed. For a given spectrum width, the variance of the velocity estimate increases directly with the increase in the overlaid power, and the estimated velocity is usable when the modulated overlaid power is less than the coherent power (i.e., equivalent to 0 dB SNR, considering the modulated power as noise). Therefore, further processing is needed to remove as much of the overlaid power as possible from the spectrum so that the ratio of the coherent signal power to the residual overlaid modulated signal power is greater than unity. This is accomplished by the notch filtering and cohering steps in the decoding algorithm. There is a certain amount of self-noise (Zrnic and Mahapatra, 1985) generated in the process of notch filtering and cohering which results in a decreased SNR. The self-noise power is a function of the code, the notch filter width, and the spectrum width. The SZ code is designed to allow the removal of the maximum amount of overlaid power, and at the same time minimize the self-noise, to improve the recovery of the velocity of the weaker signal.

2.1. Properties of the SZ phase code.

The modulation phase sequence is given by $\phi_k = n\pi k^2/M$, where M and n are integers (modulation code is $\exp\{j\phi_k\}$). If M is not divisible by n, this code has a property of zero autocorrelation for all lags except zero or multiples of M. This code was reported in a correspondence by Chu (1972). To modulate the 2nd trip signal with this code, if the 1st trip is made coherent, the transmitted phases have to be

$$\psi_k = -\sum_{p=0}^{k}\{n\pi p^2/M + \text{const.}\}\ ;\ k=0,1,2,... \qquad (2.1).$$

The constant is arbitrary and is set to zero. Another important property of the SZ modulation code is that its autocorrelation (as explained earlier) and power spectrum are independent of a shift in the code (i.e., in Eq. 2.1, k values can be from m to $m+M-1$, with arbitrary m). We refer to this code as SZ(n/M) code. Note that the symbol M is also used for representing the number of samples in the time series, and we consider values of n less than $M/2$ only. The reason for considering $1 \le n \le M/2$ is that the modulation phase code repeats after $n=M/2$, except for a shift and/or conjugation. For a given M and $n=x$, $n=iM-x$, and $n=iM+x$, the modulation codes are essentially the same for any integer i except for a conjugation and/or shift by integer multiples of $\pi/2$. The reason for choosing the parameter M to be the same as the number of samples is that M/n is the basic periodicity of the modulation code (if M/n is an integer), and for effective operation of the phase coding scheme, it is required to limit the periodicity to M or less. The indicated choice automatically limits the periodicity of the modulation code to sub-multiples of the number of samples. The number of samples available in the WSR-88D is between 44 and 66; hence, M=64 is selected as a convenient number for most of the computations in this report.

In general, for M=64 and any n, the periodicity of the modulation code can be obtained by expressing $M/n = P/q$, with all common factors between M and n removed such that q is an odd integer (i.e., equivalent to Chu's code $\phi_k = q\pi k^2/P$). P is the periodicity of the modulation code, and $4P$ is the periodicity of the switching code. Thus, it can be seen that by choosing M to be the same as the number of samples, we are restricting the periodicity of the modulation code to M or less. The autocorrelation is unity for lags in multiples of P. The spectrum of the code has only P non-zero coefficients spaced M/P coefficients apart. If a weather signal time series is multiplied by the modulation code, the spectrum of the resulting time series is a convolution of the code spectrum and the signal spectrum. If the signal spectrum is unimodal, it is easy to visualize that for n=1, the modulated spectrum is noise-like, and for n=32, it is bimodal (see Fig. 5.1, for $\pi/4$ code, Part 1 of the report). For n=1, the noise-like spectrum yields $R(1)$=0 in the mean, but there is an upper limit for the suppression of $R(1)$ by modulation, for any given realization of the signal time series with a practical number of samples. However, for n=32, the bimodal spectrum yields much better suppression of $R(1)$ because of the matching property, which is obtained for each realization. The matching property, as discussed in Part 1 of this report, is the equality of the k^{th} and $(k+M/2)^{th}$ spectral coefficient magnitudes ($|s_k|=|s_{k+M/2}|$; k=1,2,...M/2). Only the difference power ($|s_k|^2-|s_{k+M/2}|^2$) contributes to $R(1)$. As n is increased from 1 to 32, the whitening property gradually changes to matching property.

From the results presented in Part 1 of this report, it is observed that the region of recovery of v_2 in the $\{p_1/p_2;w_1\}$ space is approximately demarcated by the residual power ratio, R_p, (for definition of R_p see list of symbols, page ii) for the notch filter width used in the decoding algorithm. The $sd(\hat{v}_2)$ in the region of recovery is dictated by the overall SNR that is achieved for the weaker signal after the notch filtering and cohering steps. With a larger value for the code parameter n, better overall SNR can be achieved, but with a smaller notch width, which reduces the region of recovery of v_2 in $\{p_1/p_2;w_1\}$ space. Although earlier simulation study (Part 1 of the report) indicated the SZ(8/64) code as the best, it is not necessarily the optimum when the window and the noise effects are included. In Fig. 5.13 of Part 1 of this report, the notch width was fixed at n_w=0.75, which is not the optimum for all n values. In fact, for a given n, there is a maximum value of n_w beyond which the cohering process breaks down. This limiting value of normalized notch width can be written as

$$(n_w)_{\max} = |\, 1 - 2n/M \,| ; \quad 1 \le n \le M. \qquad (2.2)$$

This limiting value of n_w is derived from the fact that the modulation spreads the power in each of the spectral lines of the signal into M/n spectral lines separated by n coefficients, and at least two of these spectral coefficients are required for cohering the signal without the loss of the mean velocity information. Assuming that the notch width is within the maximum limit, increasing n_w

increases the region of recovery of v_2 in $\{p_1/p_2;w_1\}$ space, but at the same time, increases the standard error in $\hat{v}_2$ for a given n. Thus, the optimum n_w is the one that yields a tolerable standard error in $\hat{v}_2$ with a maximum region of recovery. For the SZ(8/64) code, n_w=0.75 yields a $sd(\hat{v}_2)$ of about 1 m s^{-1} for w_1=4 m s^{-1}, and w_2=4 m s^{-1}, without the window and noise effects. This notch width is also the maximum that we can use with this code. For larger w_2, we need to reduce the notch width to keep the $sd(\hat{v}_2)$ within the tolerable limit. If the maximum notch width (Eq. 2.2) is used for each n, and the variance of $\hat{v}_2$ is computed using simulation results, we can see that the $sd(\hat{v}_2)$ decreases with increasing n, but the extent of p_1/p_2 over which v_2 can be recovered is also reduced (Fig. 2.1).

Although the code spectrum changes drastically as n is changed from an even to an odd number (see Fig. 5.12, Part 1 of this report), the reconstructed spectrum (after the notch filtering and cohering) changes in a systematic manner, and the performance of the decoding algorithm also changes smoothly. With increasing n, the $sd(\hat{v}_2)$ and the recovery region in the $\{p_1/p_2;w_1\}$ space decreases to a minimum if the maximum admissible notch width is used in processing. When the practical effects of the window, noise, and phase error are included, the optimum n is expected to be somewhere between 8 and 16. Therefore, in this report, we examine the SZ codes with n between 8 and 16. Specifically, results are given for SZ codes with n=8,12, and 16.

In the present application, in which the phase of $R(1)$ is used for velocity computation, it suffices to have a code with $R(1)$ equal to zero; hence, any n between 1 and 63 would satisfy this requirement. For n/M=8/64, the autocorrelation is unity for lags in multiples of 8, and is zero otherwise, whereas for n/M=16/64, the autocorrelation is unity for lags in multiples of 4 and is zero otherwise. The SZ(8/64) switching and the modulation phase codes have periodicities of 32 and 8, respectively (Table 2.1a and 2.1b). In the rest of the report, the discussion and the results presented are mostly for the SZ(8/64) code. Because the SZ(12/64) and SZ(16/64) codes are similar, the discussion on these codes has been kept brief, but the final performance results are given for all three codes for comparison.

2.2. Notch filtering and cohering - SZ(8/64) code.

The spectrum of the SZ(8/64) code has exactly 8 non-zero coefficients spaced $M/8$ coefficients apart, and all are of equal magnitude (Fig. 2.2a). If the weather signal complex time series samples, e_i, are multiplied by the modulation code, C_i= exp($j\phi_i$), the resulting spectrum is shown in Fig. 2.2c. It can be seen that the weather spectrum is split into 8 identical parts having 1/8th of the original power and are separated by n coefficients (note: the magnitude of the spectral coefficient is plotted and not the power). If the spectrum width is large (Fig. 2.2d), the overlapping of the spectral coefficients produces a spectrum resembling a white noise spectrum (Fig. 2.2e).

The notch filtering process removes the overlaid stronger signal to a large extent, and the weaker signal to a lesser extent. An examination of how the notch filtering and cohering processes affect the two signals gives us a good understanding of the working of the SZ coding scheme.

First, consider the weaker 2nd trip signal. Before notch filtering, this signal spectrum is modulated by the code, C_k= exp($j\phi_k$); thus, the k^{th} coefficient of the modulated spectrum is a complex weighted sum of 8 coefficients of the unmodulated signal spectrum at $k_m=k+8m$; $m=0,1,...7$ (The DFT is cyclic; thus, if k_m is greater than M, it would be k_m-M.). The complex weights are from the modulation code. Now, assume the signal to be a d.c.; i.e., all the spectral coefficients are zero except the first coefficient. Thus, the spectrum before the notch filtering would be the spectrum of the code itself as shown in Fig. 2.3a. In the cohering process, each of the 8 non-zero coefficients is split into 8 parts and added with different phases at different locations. If any two adjacent non-zero coefficients remain after filtering, as in the case of n_w=0.75, the two components add in phase only at the first coefficient (i.e., the d.c. component), and in all other places, they add up to a smaller magnitude because of the phase difference. The spectrum after cohering is shown in Fig. 2.3c. There is a symmetry in the spectral coefficient amplitudes about the d.c. component; thus, the velocity estimate would not be biased. However, because of the remaining non-zero components, the width would be very large. It may be noted that there is no self noise generated in the process. To cohere one spectral coefficient, the maximum notch width that can be used is up to 0.86, beyond which only one non-zero coefficient remains, and the cohering process would break down. Even at n_w=0.86, the notch has to be positioned such that two adjacent non-zero coefficients are retained after notch filtering. In practice, this is not possible because the position of the notch is determined by the stronger signal spectrum, and also, the signal spectrum is not a single line but has significant width. Thus, to ensure at least two coefficients contribute in the cohering process for all the coefficients, the maximum notch width that can be used is n_w=0.75 (see Eq. 2.2). Perhaps one may operate with n_w=0.8 with some loss of coherency because the practical signals have widths of the order of 4 m s^{-1}, and one could afford to loose a few coefficients. But this will result in an increased error in the estimate because the incoherent part appears as noise.

At the outset, it appears that the SNR of the weaker signal is not affected by the notch filtering process because signal and the noise power are reduced by the same factor, $(1-n_w)$. However, in the present context of estimating the mean velocity from the phase of $R(1)$, only the part of the power contributing to $R(1)$ can be treated as the signal. How much of this cohered signal contributes to the autocorrelation, $R(1)$, can be assessed by a subtraction process in the power spectrum domain. The subtraction process is as follows: the spectral power coefficients $|s_k|^2$ and $|s_{k+M/2}|^2$ are taken at a time, and the lower of the two is subtracted from both coefficients; the process is carried out for $k=1,2,...M/2$. The subtracted part does not contribute to the $R(1)$, as can be seen from the relation between $R(1)$ and spectral coefficients (Eq. 2.7, Part 1 of this report). The subtraction process retains only three coefficients which contribute to $R(1)$: one at d.c. and one each at $+M/8$ and $-M/8$ from the d.c. component, with a reduced amplitude ($1/\sqrt{2}$ times the d.c. component; Fig. 2.3d). The signal power loss in the subtraction process is 2.19 dB. This loss factor is applicable to only the signal part because all the side bands replicate the original signal except for a complex multiplier, and it does not apply to the noise present in the signal. Thus, there is a net degradation in the SNR of the signal by 2.19 dB due to the notch filtering and cohering processes.

The signal loss computed in the previous paragraph is for a single spectral coefficient. In Fig. 2.4, the notch filtering and cohering processes are shown with a narrow width (w_2=2 m s^{-1}) weather signal. In this example, a narrow width is chosen to show the individual spectra after cohering. It is clear that the spectra shown in Fig. 2.4d and 2.4e are convolutions of the signal spectrum in Fig. 2.4a, with the spectra given in Fig. 2.3c and 2.3d, respectively. If the spectrum width is large, the signal and the two side band spectra in Fig. 2.4e overlap; thus, they cannot be clearly identified as three different spectra. Because of the overlap, the power loss in the notch filtering and cohering processes varies for each realization of the weather spectra. A simulation study with varying spectrum widths (w_2=1 to 8 m s^{-1}) yielded loss values between 1.7 dB and 3.2 dB. Further, in the overlapped portion of the spectrum, the amplitudes of the side bands vary because of the random phases; hence, the second and the third side bands do not cancel completely in $R(1)$. This can be seen in Fig. 2.4e where the second side band is not completely eliminated. The residual power from the 1st trip signal is equivalent to noise and thus further degrades the SNR.

The overlapped part of the signal and the side band spectra contribute to the variance of the velocity estimate. The mean velocity is not affected because the side bands have a symmetry about the mean velocity. Because of the overlapped spectra, there is an increase in the $sd(\hat{v}_2)$ with respect to the $sd(\hat{v}_2)$ of the original signal using the autocovariance algorithm.

Now, we consider the notch filtering and cohering processes with respect to the 1st trip signal. Assume that the 2nd trip signal is absent, and the 1st trip signal has a Gaussian shape. The effect of notch filtering and cohering processes on the 1st trip signal is illustrated with an example of a simulated weather signal of w_1=6 m s^{-1} (Fig. 2.5a). Since the 1st trip signal is coherent before notch filtering, and the notch is centered on the mean velocity, the residual power is equal to (p_1/R_p), where

$$R_p = 1 \,/\, [1 - \mathrm{erf}\{n_w v_a/(w_1\sqrt{2})\}] \qquad (2.3)$$

is the residual power ratio (i.e., the ratio of the total power, p_1, to the residual power after notch filtering). Note that the expression (2.3) is for a Gaussian spectrum and not for the sampled signal spectrum with finite number of samples, but it is used here because it is a fairly good approximation up to n_w=0.9. The cohering of the 2nd trip signal after the notch filtering, modulates the 1st trip residual power; the modulation code in this case is the complex conjugate of C_k. If the notch width is zero, the spectrum would appear more like a white noise for large widths (as in Fig. 2.2e) because each coefficient is a complex weighted sum of 8 coefficients of the original spectrum before modulation. However, with increasing notch width, less coefficients are added, resulting in less of the white noise part and more of the colored noise. At the same time the match property improves with increasing notch width, and perfect matching is obtained for n_w = 0.875 because the overlap is zero. The match property is $|s_k|=|s_{k+M/2}|$, or the left and right halves of the spectrum have the same envelope (see eq. 5.4 of part-1 of this report). But we cannot use this notch width because of the reasons stated earlier. The maximum usable n_w=0.75 results in a less than perfect match. The matched part and the white noise part do not contribute to the autocorrelation, $R(1)$ and, thus, would not affect the mean velocity. With notch filtering, only the tail ends of the spectrum are retained; the power is suppressed by 42 dB by the notch filter of width n_w=0.75. In the cohering step, the residual signal is phase modulated resulting in a spread of the power into 8 identical spectra having 1/8 of the power. The matching property can be clearly seen in the spectrum shown in Fig. 2.5c. The part of the power that contributes to $R(1)$ can be obtained by the subtraction process, which accounts for another 8 dB of suppression (Fig. 2.5d). The subtraction is used only to compute the suppression of overlaid power. It is not used in the decoding algorithm (described in section 5.4) which automatically accomplishes equivalent suppression in computing $R(1)$. The suppression of overlaid power is a function of the spectrum width w_1 and the notch width.

To get an idea of how much of the residual 1st trip noise in $R(1)$ is suppressed, a simulation study was conducted with only the 1st trip signal subjected to the notch filtering, cohering, and subtracting processes. The ratio of the residual power before and after the subtraction, μ, as a function of the notch width is shown in Fig. 2.6 for w_1=4 m s^{-1}. The mean value of μ remains fairly constant at 3.6 dB up to about n_w = 0.55 and then starts increasing; it is about 6.5 dB (mean value; a ±2 dB variation is observed in the simulation) at n_w = 0.75. This behavior is because of the improvement in the matching property for larger notch widths. The variation of the μ with the spectrum width is plotted in Fig. 2.7 for the selected n_w=0.75, which indicates that the μ increases with decreasing spectrum width because of the improvement in the matching property.

When both signals are present, there is a loss of the 2nd trip signal of about 7.7 to 9.2 dB (~8.5 dB, of which 6 dB is caused by the notch filter) with the notch filtering and cohering processes (as discussed earlier in this section), and there is a reduction in the 1st trip residual signal (noise-like) by about 5 to 8 dB (~6.5dB) in addition to the reduction by the residual power ratio, R_p. The net improvement in SNR is (R_p+6.5-8.5) dB on average, at a n_w=0.75 and w_1=4 m s^{-1}, which is about 2 dB less than the R_p. Approximate overall SNR for n_w=0.75 can be written as

$$\text{overall SNR} = (10^{-0.25}/4) \;/\; [0.25/\text{SNR}_2 + 10^{-0.65}\, p_1/(p_2 R_p)] \qquad (2.4)$$

The improvement in SNR is made possible by the notch filtering and cohering processes because the residual power of the stronger signal decreases much faster than the weaker signal power, as a function of notch width. It must be noted here that (2.4) includes the SNR degradation due to the signal power loss alone in the processing. There is some amount of noise generated due to the overlapped part of the spectrum after notch filtering and cohering processes, as indicated earlier in this section, which is a function of the spectrum width of the signal. Thus, in practice, the overall SNR would be lower than that given by (2.4).

2.3. Deconvolution procedure for spectrum width estimation - SZ(8/64) code.

So far, not much attention has been given to the spectrum width estimate of the weaker signal. The six undesirable side bands generated by the notch filtering and cohering processes produce a bias error in the spectrum width $\hat{w}_2$. The estimated width is always larger than the actual width.

An examination of the spectrum after the notch filtering and cohering processes (Fig. 2.4d) shows that the spectrum consists of the actual signal and three symmetrically placed side bands on either side, each with different amplitudes. These are shifted versions of the signal (with shifts in multiples of $M/8$ coefficients) multiplied by a complex number. Therefore, there is an amplitude change and phase shift associated with the multiplier. The magnitude spectrum appears as a convolution of the spectrum of the 2nd trip signal with the code spectrum obtained after notch filtering and cohering (see Fig. 2.3c), but it is not exactly a convolution because of the notch filter. A deconvolution in the complex domain is not possible because the embedded notch filtering process makes the matrix associated with convolution singular. In fact, the rank of the associated matrix is determined by the notch width and is given by $(1-n_w)M$; for example, the rank of the MxM matrix is $M/4$ for $n_w = 0.75$. An alternative is to deconvolve using only the magnitudes, which does not result in exactly the same signal spectrum but is fairly close to it. For narrow widths (w_2<2 m s $^{-1}$), the restored shape is almost like the original, but for larger w_2, the shape is not reproduced exactly. An example of the magnitude deconvolution is given in Fig. 2.8, for a narrow spectrum (w_2=2 m s^{-1}); the spectrum at different stages of the processing is shown. The power levels indicated on the figure are nominal values but can vary with each realization. For larger widths, the spectrum overlap combined with in-exact magnitude domain deconvolution produces a variation in power levels; it is about 6±1 dB after notch filtering, and about 0.5 to 4 dB additional loss in the deconvolution, depending on the spectrum width. The spectrum width is computed using this restored spectrum. A simulation study indicates that there is no bias in the width estimate, but the variance is larger than that obtained with an autocovariance algorithm in the absence of the overlaid signal. Therefore, this procedure is adopted for width estimation in the stand-alone version of the SZ decoding algorithm (SZ-1 algorithm).

The convolution matrix (magnitude only) is a MxM real matrix. The first row vector of the convolution matrix is obtained as follows. The spectrum of the modulation code is notch filtered and cohered (Fig. 2.3c), and then the normalized magnitude spectrum is computed (total power is normalized to unity). The notch filter width is the same as that used in the SZ decoding algorithm, but the position can be anywhere because the magnitude spectrum is independent of the position; only the phases are dependent on the position. All other rows of the matrix are obtained by shifting cyclically the first row by (n-1) coefficients to the right for the n^{th} row. The matrix, thus obtained, is inverted to get the deconvolution matrix. The deconvolution process consists of pre-multiplying the magnitude spectrum (Mx1 -column matrix), obtained after the notch and cohere processes, by the deconvolution matrix. The result is a column matrix which is the recovered magnitude spectrum. The deconvolution matrix is pre-computed and is input to the SZ decoding algorithm.

The deconvolution procedure allows us to cohere all the side band power which otherwise would correspond to a 6 dB loss for n_w=0.75. However, since the deconvolution is carried out in the magnitude domain, the power in the spectrum is not conserved. There is a loss of the signal which is a function of the spectrum width. The loss is about 0.5 dB for w_2=1 m s^{-1} and can be as large as 4 dB for w_2=8 m s^{-1}. For a median width of 4 m s^{-1}, the loss is about 2.5 dB. It is shown later that there is a corresponding loss in the residual stronger signal also by about the same factor (see 2 paragraphs below). Thus, the net improvement is still 6 dB.

To evaluate the efficacy of the deconvolution process in recovering the weaker signal spectrum, especially the spectrum width, a simulation study was carried out with the SZ(8/64) coded 2nd trip signal subjected to the notch filtering and cohering processes. The 1st trip signal is assumed to be absent. Fig. 2.9a is a scatter plot of the spectrum width recovered after notch filtering, cohering, and deconvolution processes, versus the input width to the simulation program. The mean and the standard deviation is also shown on the plot. In generating this plot, the following parameters are used: v_a=32 m s^{-1}, M=64, SNR$_2$=40 dB, n_w=0.75, and v_2 is randomly selected within ±28 m s^{-1}. The spectrum width is computed using the ratio of $R(0)/R(1)$ (Eq. 2.6 of part 1 of this report). Compare this with Fig. 2.9b which is a similar plot with the same parameters for the simulation, but the width is estimated from the uncoded time series using the autocovariance algorithm. The standard deviation of the width estimate is increased to some extent by the notch filtering, cohering, and magnitude domain deconvolution processes.

It is important to examine the effect of the deconvolution procedure on the 1st trip residual signal, especially the effect on the residual power and the nature of the residual spectrum after deconvolution. In Fig. 2.10, a series of spectra of the 1st trip signal are shown at different stages of processing. The 2nd trip signal is assumed to be absent, and a large spectrum width of 6 m s^{-1} is chosen for the 1st trip signal so that the residual power after notch filtering is substantial for demonstration purposes. After the notch filtering and cohering processes, the spectrum has an appearance of white noise but has a matching property because of the systematic code. The last spectrum in the figure is after the deconvolution. It can be seen that the spectrum still has the appearance of a white noise, but the power is less by about 7 dB. The power loss is because of the magnitude domain deconvolution process. The power levels shown on the figure are nominal values and may vary for each realization of the spectrum. The reduction in the residual power is a function of the spectrum width. The loss and its variation due to the deconvolution procedure is larger for larger spectrum widths. A simulation study yielded loss values between 3 dB and 8 dB for widths between 1 and 8 m s^{-1}. The loss of the residual power improves the SNR; however, this is partly off-set by the loss of weaker coherent signal as well, as discussed earlier (see 2 paragraphs above). On the other hand, if the residual power loses its white noise property or the matching property due to deconvolution, then it can introduce bias error in the velocity estimate of the recovered signal. Again, a simulation study using different widths indicates that the mean velocity of the residual spectrum is zero; thus, the deconvolution does not introduce bias error in $\hat{v}_2$. However, the variance of the mean velocity estimate increases due to the deconvolution process because the reconstructed magnitude spectrum is not exact. Therefore, the deconvolution step is used only for spectrum width computation, and the velocity is estimated before the deconvolution.

Now, if both 1st and 2nd trip signals are present in the spectrum, the magnitude domain deconvolution procedure would affect the two signals in a somewhat similar manner, but the composite result may vary because the magnitude spectrum is the magnitude of the vectorial sum of the two spectra. Fig. 2.11 demonstrates the recovery of the weaker 2nd trip signal in the presence of a strong 1st trip signal. The parameters used are shown in the figure. It can be seen that the recovered spectrum (Fig. 2.11e) does not have exactly the same shape as the original 2nd trip signal (Fig. 2.11b) but has nearly the same mean velocity and width.

Table.2.1a. Modulation phase code sequence for *n/M*=8/64 in degrees.

k	ϕ_k	k	ϕ_k	k	ϕ_k	k	ϕ_k
0	0.0	16	0.0	32	0.0	48	0.0
1	22.5	17	22.5	33	22.5	49	22.5
2	90.0	18	90.0	34	90.0	50	90.0
3	-157.5	19	-157.5	35	-157.5	51	-157.5
4	0.0	20	0.0	36	0.0	52	0.0
5	-157.5	21	-157.5	37	-157.5	53	-157.5
6	90.0	22	90.0	38	90.0	54	90.0
7	22.5	23	22.5	39	22.5	55	22.5
8	0.0	24	0.0	40	0.0	56	0.0
9	22.5	25	22.5	41	22.5	57	22.5
10	90.0	26	90.0	42	90.0	58	90.0
11	-157.5	27	-157.5	43	-157.5	59	-157.5
12	0.0	28	0.0	44	0.0	60	0.0
13	-157.5	29	-157.5	45	-157.5	61	-157.5
14	90.0	30	90.0	46	90.0	62	90.0
15	22.5	31	22.5	47	22.5	63	22.5

Table.2.1b. Switching phase code sequence for *n/M*=8/64 in degrees.

k	ψ_k	k	ψ_k	k	ψ_k	k	ψ_k
0	0.0	16	180.0	32	0.0	48	180.0
1	22.5	17	-157.5	33	22.5	49	-157.5
2	112.5	18	-67.5	34	112.5	50	-67.5
3	-45.0	19	135.0	35	-45.0	51	135.0
4	-45.0	20	135.0	36	-45.0	52	135.0
5	157.5	21	-22.5	37	157.5	53	-22.5
6	-112.5	22	67.5	38	-112.5	54	67.5
7	-90.0	23	90.0	39	-90.0	55	90.0
8	-90.0	24	90.0	40	-90.0	56	90.0
9	-67.5	25	112.5	41	-67.5	57	112.5
10	22.5	26	-157.5	42	22.5	58	-157.5
11	-135.0	27	45.0	43	-135.0	59	45.0
12	-135.0	28	45.0	44	-135.0	60	45.0
13	67.5	29	-112.5	45	67.5	61	-112.5
14	157.5	30	-22.5	46	157.5	62	-22.5
15	180.0	31	0.0	47	180.0	63	0.0