

ERRATA AND SUPPLEMENTS¹
for the third and fourth printings of
Doppler Radar and Weather Observations, Second Edition-1993
Richard J. Doviak and Dusan S. Zrnić
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Page	Para.	Line	Remarks: Paragraph 0 is any paragraph started on a previous page that carries over to the current page. Section heading are not counted as a line, but equations are so counted. A sequence of dots is used to indicate a logical continuation to existing words in the textbook (e.g., see errata for p.14; or that for Fig. 3.3 caption; etc.)
xxi		θ	modify definition to read: “is the zenith angle (Fig. 3.1); also the angle from the axis of a circularly symmetric beam (p. 34); also potential energy
14	2	2	change to read: “...index $n = c/v$ with height (or, because the relative permeability μ_r of air is unity, on the change of relative permittivity, $\epsilon_r = \epsilon/\epsilon_0 = n^2$, with height).
15	1	7	insert the reference (Born and Wolf, 1964, p. 87)
17	1	2-6	line 2, change “ $T=300$ K” to “ $T=290$ K”; line 4, change this equation to read: $N \approx 0.268 \times (10^3 + 1.66 \times 10^2) \approx 312$; and line 6 change “1.000300” to “1.000312”.

1 These errata are periodically updated and posted on NSSL’s website at www.nssl.noaa.gov. In the “Quick Links” box select “Publications” to open the page to select “Recent Books” to find the book and listed Errata for the 3rd and 4th printings. Following these errata are supplements that clarify or extend the book text. The authors appreciate comments from users of the book; these comments have led to updated errata and supplements that help to keep the book correct and current.

2 The Dover Edition is a copy of the 1st and 2nd printings (not the 3rd and 4th printings as stated in the preface to the Dover edition; Oops!) of the 1993 edition of the Academic Press publication. Errata to the 1st and 2nd printings are also at the same NSSL website. Readers of the Dover edition need to refer to both errata to have all corrections.

- 23 Eq. (2.29) change $n(h)$ to $N(h)$ and n_s to N_s
- 0 10 change “refractive index” to “refractivity N ”
- 30 2 9 replace the italicized “*o*” from the first entry of the word “oscillator” with a regular “o”, but italicize the “*o*” in the second entry of the word “oscillator”
- 3 7 delete the parenthetical phrase
- 34 Eq. (3.2) replace D with D_a
- 2 3 change “intensity” to “power density”
- 6 change to read: “defines the maximum directivity of the transmitting antenna.”
- 35 1 9 at the end of the last sentence add: with origin at the scatterer.
- 2 10 the equation on this line should read:
- $$\sigma_b = \sigma_{bm} \left(1 - \sin^2 \psi / \sin^2 \theta\right)^2 \cos^4 [(\pi / 2) \cos \theta] / \sin^4 \theta$$
- Eq. (3.6) and on the line after this equation, change “ K_m ” to “ K_w ”
- 36 0 7 delete “ $|K_m|^2 \equiv$ ”
- 9 change the end of this line to read: “Ice water has a $|K_w|^2 \equiv$ ”
- Fig.3.3 caption revise to read: “.....(a) Liquid water; the square of the complex refractive index m^2 (at 0° C) is.... “
Furthermore, the sign of j everywhere in this caption needs to be changed from + to -.
- 40 Eq. (3.14b) replace subscript “m” with “w”
- 46 1 7 change to read: “..., and that g accounts for losses in the antenna, the radome, and in the transmission lines from the antenna to the point where P_t and P_r are measured.
- 47 Table 3.1 1) change title to read: “The *next* generation *radar*, NEXRAD (WSR-88D), Specifications”
2) change “Beam width” to “Beamwidth”

3) change footnote *b* to read: “Initially the first several radars transmitted circularly polarized waves, but now all transmit linearly polarized waves”.
 4) change footnote *c* to read: “Transmitted power, antenna gain (including radome loss), and receiver noise power are referenced to the antenna port.

- 48 0 4, 5 change 2×10^{-7} to 4.2×10^{-7} , and 6.3 to 7.3.
- 61 Eq. (3.40b) place \pm before v_a
- 0 14 last line change to “velocity limits (Chapter 7).”
- 68 3 7-8 change to read as: “...is thus the expected power $E[P(\tau_s)]$.”
- 4 1 start this sentence as “ $E[P(\tau_s)]$ does not change...”
- 69 0 6, 10 change $\bar{P}(\tau_s)$ to $E[P(\tau_s)]$.
- 71 Eqs. (4.4a,b) insert $(1/\sqrt{2})$ in front of the sum sign in each of these equations
- 3 6 replace “p. 418” with “p. 498”.
- Eq. (4.6) delete the first “2”
- 72 0 4 change to: “.and a mean or expected value $E[P(\tau_s)] = 2\sigma^2$.”
- 2 1 change $\bar{P}(\tau_s)$ to $E[P(\tau_s)]$
- 3 remove footnote and its symbol appended to $E[P_i]$
- 73 Eq. (4.11) change “ $\bar{P}(\mathbf{r}_0)$ ” to “ $E[P(\mathbf{r}_0)]$ ”.
- Eqs. (4.12), (4.14), (4.16): change “ $\bar{P}(\mathbf{r}_0)$ ” to “ $E[P(\mathbf{r}_0)]$ ”.
- Eqs. (4.14), (4.16) change η to $\eta(\mathbf{r}_0)$, and change l^2 to $l^2(\mathbf{r}_0)$
- 75 1 6 change to “ $G(0) = 1$ ”
- 2 16 change (4.12) to (4.14)
- 18 change “ $\bar{P}(\mathbf{r}_0)$ ” to “ $E[P(\mathbf{r}_0)]$ ”.
- 76 Fig.4.5 change second sentence in caption to read: “The broad arrow indicates

- sliding of....”
- 77 0 12 change “mean” to “expected”
- 13 change “ $\bar{P}(\tau_s)$ ” to “ $E[P(\tau_s)]$ ”
- Eq. (4.21) change “ $\bar{P}(\tau_s)$ ” to “ $E[P(\tau_s)]$ ”
- Eq. (4.22) delete $\equiv |W(r)|^2$
- 78 Fig. 4.7 change the argument ‘ r ’ in $|W(r)|^2$ to ‘ $\frac{c\tau_s}{2} - r$ ’
- 79 2 3 change to: “...a scatterer at r has the approximate range-dependent...”
- 8-9 change to: “...the weighting function about its peak at any range $r = r_0$.”
- 82 Eq. (4.34) change “ $P(\bar{\mathbf{r}}_0)$ ” to “ $E[P(\mathbf{r}_0)]$ ”.
- Eq. (4.35) change “ $\bar{P}(\text{mw})$ ” to “ $E[P(\text{mw})]$ ”
- 1 9 should read: “.. is the *reflectivity factor* of spheres.”
- Eq. (4.38) subscript “ τ ” should be the same size as in Eq.(4.37).
- 84 Eqs. (4.39), (4.43) change “ $\bar{P}(\mathbf{r}_0)$ ” to “ $E[P(\mathbf{r}_0)]$ ”.
- 85 0 4 change “ $\bar{P}(\mathbf{r}_0)$ ” to “ $E[P(\mathbf{r}_0)]$ ”.
- Problem 4.1 change “ \bar{P} ” to “ $E[P]$ ” in two places.
- Eq. (5.40) change the arguments \mathbf{r}_0 and \mathbf{r}_1 of W_s to r_0 and r_1 , and change $l^2(\mathbf{r}_1)$ to $l^2(r_0, \mathbf{r}_1)$
- 108 1 1 change “stationary” to “steady”
- 1 11 change “ $d\bar{P}$ ” to “ $E[dP]$ ”.
- Eq. (5.42) change “ $d\bar{P}(v)$ ” to “ $E[dP(v)]$ ”

- 15 change “ $\bar{P}(\mathbf{r}_0, \nu)$ ” to “ $E[\Delta P(\mathbf{r}_0, \nu)]$ ”
- Eq. (5.43) change “ $\bar{P}(\mathbf{r}_0, \nu)$ ” to “ $E[\Delta P(\mathbf{r}_0, \nu)]$ ”
- 1 after Eq. (5.43) insert “ $\Delta P(\mathbf{r}_0, \nu)$ is the differential power from all the elemental volumes having the Doppler velocity ν centered in the interval $d\nu$.”
- 3 2-3 change to read: “...by new ones having different spatial configurations, the estimates $\hat{S}(\mathbf{r}_0, \nu)$ of ...”
- 109 Eq. (5.45) change “ $\bar{P}(\mathbf{r}_0)$ ” to “ $E[P(\mathbf{r}_0)]$ ”
- change footnote “4” to read: “The overbar on a variable denotes a spatial (i.e., volume) average, whether or not the average is weighted.”
- 113 1 1-4 change to read: “Assume scatterer velocity is the sum of steady $v_s(\mathbf{r})$ and turbulent $v_t(\mathbf{r}, t)$ wind components. Each contributes to the width of the power spectrum (even uniform wind contributes to the width because radial velocities $v_s(\mathbf{r})$ vary across V_6 ; steady wind also brings new....”
- 2 10 delete the sentences beginning on line 10 in paragraph 2 with “Furthermore, we assume...” and ending in paragraph 3, line 3 with “...scatterer’s axis of symmetry.”
- 114 0 5 insert and modify after “...initial phases ϕ_i ”: Here expectations are made over an ensemble of scatterer configurations (see supplement to Section 10.2.2). Because $R(0)$ is proportional to the expected power $E[P(\mathbf{r}_0)]$, and because
- $$E[P(\mathbf{r}_0)] = \sum_k E_\sigma [\sigma_{bk}] I(\mathbf{r}_0, \mathbf{r}_k) \quad (5.59c)$$
- [i.e., from Eq. (4.11)], where $E_\sigma [\sigma_{bk}]$ is the expected backscattering cross section σ (expectations computed over the ensemble of σ) of the k^{th} hydrometeor, it follows that $R_k(0)$ is proportional to $E_\sigma [\sigma_{bk}]$ and”
- 2 2-4 modify to read: “...mechanisms in Eq. (5.59b) act through product terms. Furthermore, the k th scatterer’s radial velocity v_k can be expressed as the sum of the velocities due to steady and turbulent winds that move the

scatterer from one range position...”

6-9 delete these lines and replace with:

“...Eq. (5.59a), the velocities $v_s(\mathbf{r})$ and $v_t(\mathbf{r}, t)$ associated with steady and turbulent winds can each be placed into separate exponential functions that multiply one another. Thus the expectation of the product can be expressed by the product of the exponential containing $v_s(\mathbf{r})$ and the expectation of the exponential function containing $v_t(\mathbf{r}, t)$; these exponential functions are correlation functions. The Fourier transform of $R(mT_s)$, giving the composite spectrum $S(f)$, can then be expressed as a convolution of the spectra associated with each of the three correlation functions. There are other de-correlating mechanisms (e.g., differential terminal velocities, antenna motion, etc.,) that increase the number of correlation functions and spectra to be convolved. It is shown that, ...”

115 3 1 “R” in “ R_k ” should be italicized to read “ R_k ”

7 change “Eq. (5.59b)” to “Eq. (5.59a)”

14 change these lines and Eqs. (5.64) to read: “Because the correlation coefficient can be related to the normalized power spectrum $S_n(f)$ by using Eq. (5.19), and because the Doppler shift $f = -2v/\lambda$, $\rho(mT_s)$ can be expressed as

$$\rho(mT_s) = \int_{-\lambda/4T_s}^{\lambda/4T_s} \frac{2}{\lambda} E \left[\hat{S}_n^{(f)}(-2v/\lambda) \right] e^{-j4\pi mT_s/\lambda} dv = \int_{-v_a}^{v_a} E \left[\hat{S}_n(v) \right] e^{-j4\pi mT_s/\lambda} dv \quad (5.64)$$

116 0 1-4 change these lines to read: where $\hat{S}_n^{(f)}(-2v/\lambda)$ is the estimated normalized power spectrum in the frequency domain, $\hat{S}_n(v)$ is the estimated normalized power spectrum in the Doppler velocity domain, and these two power spectra are related as

$$\hat{S}(v) = \frac{2}{\lambda} \hat{S}^{(f)}(-2v/\lambda). \quad (5.65)$$

By equating Eq. (5.63) to Eq. (5.64), and assuming all power is confined within the Nyquist limits, $\pm v_a$, it can be concluded that

$$p(v) = E_v \left[\hat{S}_n(v) \right], \quad (5.66)$$

1 1-7 change to read: “where the expectation E_v is taken over the ensemble of velocity fields (for additional explanation of E_v see supplement for Section

10.2.2). Thus, for homogeneous turbulence, at least homogeneous throughout the resolution volume V_6 , the *expected* normalized power spectrum is equal to the velocity probability distribution. Moreover, it is independent of reflectivity and the angular and range weighting functions.

- 2 15-21 the two sentences beginning with “Because the cited spectral” should be modified to read: “Contrary to accepted usage, the estimates of the second central moment σ_v^2 of the Doppler spectrum is not necessarily the sum of the second central moments of individual spectral broadening mechanisms. It has been shown (Fang and Doviak, 2008)³ the variance associated with shear and antenna motion cannot be separated into a sum of second central moments, and moreover there is an additional term associated with the cross product of turbulence and shear. But if turbulence, hydrometeor oscillation/wobble, and terminal velocities are locally homogeneous (i.e., statistically homogeneous), and estimates are averaged (i.e., spatial and/or temporal), the expected σ_v^2 can be expressed as the sum

$$\sigma_v^2 = \sigma_a^2 + \sigma_{sa}^2 + \sigma_o^2 + \sigma_t^2 + \sigma_d^2, \quad (5.67)$$

A rigorous derivation of the spectrum width equation, for non-homogeneous conditions and nearly horizontal beams so terminal velocity is negligible, is given by Eq. (B.13) in Fang and Doviak (2008). Because Doppler shifts associated with terminal velocities of hydrometeors is independent of wind, the second central moment (i.e., σ_d^2) due to variance in terminal velocities (Section 8.2) of different size hydrometeors has been added to the equation given by Fang and Doviak (2008).

In (5.67), σ_a^2 is related to changes in weather signal sample correlation because beam location for each sample changes as the beam azimuthally scans at a rate of α (radian/sec). That is, weather signal samples from different resolution volumes (V_6) are not as well correlated as those from the same V_6 . The term σ_{sa}^2 is the shear contribution (i.e., from radial, elevation, and azimuth shear) which depends on α because the azimuth shear contribution increases due to an effectively larger azimuthal beam width (Section 7.8). σ_o^2 is due to changes in orientation or vibration of hydrometeors, and σ_t^2 is due to turbulence.”

- 2 21-23 delete the last sentence of this paragraph

³ Fang, M., and R. J. Doviak, 2008: Coupled Contributions in the Doppler Radar Spectrum Width Equation. *J. Atmos. Oceanic Technol.*, **25**, 2245-2258.

- 117 1 7 at the end of the sentence, "...to the beam center.", begin a new paragraph by modifying the lines following the sentence to read: "If there is no radial velocity shear, and if the antenna pattern is Gaussian.....width θ_1 , and the antenna rotates at an..."
- 2 1 change this line to read: "Assume the beam is stationary. We shall prove that the term $\sigma_{s\alpha}^2 \rightarrow \sigma_s^2$ is composed of three...."
- 4-7 modify these lines to read: "where the terms are due to shear of v_s , the radial component of steady wind, along the three spherical coordinates at r_0 . In this coordinate system (5.70) automatically includes..."
- 9 change to read: "the so-called beam-broadening term;...."
- 117 3 replace the text in this paragraph up to and including equations to Eq. (5.75) with:
 "Spherical coordinate shears of radial velocity v_s of steady wind can be directly measured with the radar and it is natural to express σ_s^2 in terms of these shears. If $\theta_1 \ll 1$ (radian) and $\theta_0 \gg \theta_1$, and angular and radial shears are uniform, v_s can be expressed as

$$v_s - v_0 \approx k_r(r - r_0) + k_\theta r_0(\theta - \theta_0) + k_\phi r_0(\phi - \phi_0) \sin \theta_0 \quad (5.71)$$

where

$$k_r \equiv \frac{dv_s}{dr}, \quad k_\theta \equiv \frac{1}{r_0} \frac{dv_s}{d\theta}, \quad k_\phi \equiv \frac{1}{r_0 \sin \theta_0} \frac{dv_s}{d\phi} \quad (5.72)$$

are angular and radial shears of v_s . Angular shear in units of s^{-1} is defined as the Doppler (radial velocity) change per differential arc *length* (e.g., $r_0 d\phi \sin \theta_0$).

Angular shear can be non-zero even if Cartesian shears are zero. For example, if the Cartesian components of wind are constants u_0, v_0, w_0 , the angular and radial shears are

$$\begin{aligned}
k_r &= 0; \\
k_\theta &= \frac{1}{r_0} (u_0 \sin \varphi_0 + v_0 \cos \varphi_0) \cos \theta_0 - \frac{1}{r_0} w_0 \sin \theta_0; \\
k_\varphi &= \frac{1}{r_0} (u_0 \cos \varphi_0 - v_0 \sin \varphi_0)
\end{aligned} \tag{5.73}$$

Assume V_6 is sufficiently small such that reflectivity and the angular and radial shears are practically uniform across V_6 and the weighting function is product separable and symmetric about \mathbf{r}_0 . Then we can substitute Eq. (5.71) into Eq. (5.51) to obtain

$$\sigma_s^2(\mathbf{r}_0) = \sigma_{s\theta}^2 + \sigma_{s\phi}^2 + \sigma_{sr}^2 = k_\theta^2 r_0^2 \sigma_\theta^2 + k_\phi^2 r_0^2 \sigma_\phi^2(\theta_0) \sin^2 \theta_0 + k_r^2 \sigma_r^2. \tag{5.74}$$

Because lines of constant ϕ converge at the vertical, the second central moment $\sigma_\phi^2(\theta_0)$ of the two-way azimuthal radiation pattern is a function of zenith angle θ_0 . That is, $\sigma_\phi^2(\theta_0) = \sigma_\theta^2 / \sin^2 \theta_0$, where σ_θ is the intrinsic beamwidth for a circularly symmetric beam. The intrinsic beam width is that measured in a spherical coordinate system in which the polar axis is along the beam axis. The intrinsic beamwidth is invariant with respect to the direction of the beam. On the other hand $\sigma_\phi^2(\theta_0)$, measured in the spherical coordinate system centered on the radar but in which the polar axis is vertical (i.e., the so called radar coordinate system), increases with decreasing θ_0 . σ_r^2 is the second central moment of $|W(r)|^2$. For circularly symmetric Gaussian radiation patterns having an intrinsic beamwidth θ_1 ,

$$\sigma_\theta = \frac{\theta_1}{4\sqrt{\ln 2}}; \quad \sigma_\phi(\theta_0) = \frac{\theta_1}{4\sqrt{\ln 2}} \frac{1}{\sin \theta_0} \tag{5.75}$$

118 0

after Eq. (5.76) add: “The above derivation ignored effects of beam scanning during the dwell time MT_s . If the beam scans at an azimuth rate α , it can be shown (5.74) should be written as

$$\sigma_{sa}^2 = k_\theta^2 r_0^2 \sigma_\theta^2 + k_\phi^2 r_0^2 \sigma_{\phi e}^2(\alpha, \theta_0) \sin^2 \theta_0 + k_r^2 \sigma_r^2 \tag{5.77}$$

where $\sigma_{\phi e}(\alpha, \theta_0) = \theta_{1e}(\alpha) / 4 \sin(\theta_0) \sqrt{\ln 2}$, is the azimuthal beamwidth effectively broadened by antenna rotation during MT_s , and $\theta_{1e}(\alpha)$ is the effective one-way half-power azimuthal width, a function of αMT_s (Fig. 7.25).

124	3	1, 5	change “video” to “voltage” and change “signal” to “voltage”.
125	1	1	replace “average” with “expected”
	Eq. (6.5)		append to this equation the footnote: “In chapter 5 ρ is the complex correlation coefficient. Henceforth it represents the magnitude of this complex function.”
	4	5	remove the overbar on P , S , and N
126	0	1	change to read: “power estimate \hat{P} is reduced.....variance of the P_k ..”
	3	2-4	the second sentence, modified to read, “The P_k values of meteorological interest...meeting this large dynamic range requirement”, should be moved to the end of the paragraph 1
		5	change “ \bar{P} ” to “ S ”.
127	0	1-2	remove the overbar on P in the three places
	3	1	remove the overbar on Q
		8	delete the citation “(Papoulis, 1965)”
128	1	8	change “unambiguous” to “Nyquist”
	2	4-7	rewrite the second and third sentences after Eq. (6.12) as: “The variance of the estimates \hat{S} , each obtained by averaging M un-weighted signal power samples, is calculated using the distribution given by Eq. (4.7) to calculate the single sample variance σ_Q^2 in Eq. (6.9) (in using Eq. (4.7) we set $P \rightarrow \hat{S}$ because noise power is assumed to be zero); this gives $\sigma_Q^2 = S^2$. Thus the variance of the M sample average is, from Eq.6.10, S^2 / M_I where M_I is calculated from Eq. (6.12).”
	3	1-2	change to read “To estimate S in presence of receiver noise, we need to subtract.....”
		4-9	remove overbars on P , N , and S
129	0	5-6	change last sentence to read: “....then the number of independent samples can be determined using an analysis similar to.....”

130	Table 6.1		add above “ Reflectivity factor calculator ” the new entry “ Sampling rate ”, and in the right column on the same line insert “0.6 MHz”. Under “ Reflectivity factor calculator ”, “Range increment” should be “0.25 km” and not “1 or 2 km”. But insert as the final entry under “ Reflectivity factor calculator ” the entry “Range interval Δr ”, and on the same line insert “1 or 2 km” in the right column.
134	1	4	change to: “.....are independent, $\text{var}(\hat{v})$, obtained directly from (6.21), is Eq. (6.22b) change approximate sign to equal sign
136	footnote		change to read: “To avoid occurrence of negative \hat{S} , only the sum in Eq. (6.28) is used but it is multiplied with $\hat{S}\hat{N}R / (\hat{S}\hat{N}R + 1)$ ”
137	2	1	delete “($\sigma_{v_m} > 1 / 2\pi$)”
142	Eq. (6.42)		place a caret
150	1	4	rewrite line 4 to read: “...shift $\psi_d = -2T_s v(k + \bar{k}_v)$ (\bar{k}_v is the <i>path averaged</i> wavenumber increment associated the vertically polarized wave propagating through precipitation along the propagation path) and the two-way total differential phase,”
155	3	3	in Section 6.8.5 line 3, change “Because” to “If”
160	2	6	change “unambiguous velocity ” to “Nyquist velocity”
171	0	3	T_s should be T_2
173	0	1	change to read: “...velocity interval $\pm v_m$ for this....” Eq. (7.6b) place \pm before v_m
	3	9-10	this should read: “...the desired unambiguous velocity interval. An unambiguous velocity interval $v_m = \dots$ ”
		11	change “unambiguous” to “Nyquist”
182	Eq. (7.12)		$W_i W_{i+1}$ should be $W_i W_{i+l}$

197	1	1	“though” should be “through”
	2	4	“Fig.3.3” should be “Fig.3.2”
200	Fig.7.28		Along the abscissa, change AZIMUTH ($^{\circ}$) to ELEVATION ($^{\circ}$), and in the caption delete the parenthetical phrase at the end of the caption. The second sentence should read: “Sidelobes with radome are specified to be below the dashed lines.
201	0	2	“Norma” should be “Norman”
	Eq. (7.36)		change “ $\bar{P}(r_0)$ ” to “ $E[P(\mathbf{r}_0)]$ ”, and the upper integration limit to 2π
213	1		change this paragraph to read: The Marshall-Palmer (M-P) data extend over a relatively short range of drop sizes (Fig. 8.3a). Earlier measurements (Laws and Parsons, 1943; Fig. 8.3b) that span a much larger range of drop diameters show that the drop size distributions (DSDs) at small drop diameters do not necessarily converge to a constant N_0 as suggested by Marshal and Palmer. The large increase in drop density at smaller drop diameters is also seen in the theoretical steady-state distributions derived by Srivastava (1971). But other measurements (citation?) show DSD decreasing at smaller drops. Thus the one free parameter (i.e., Λ) of an exponential DSD is at best a rough estimate of the true DSD.
222	Eq. (8.18)		the differential “dD” on the left side of Eq.(8.18) must be moved to the end of this equation.
228	1	2-3	change to read “...(Smith, 19084). Assuming Rayleigh scatter, the radar equation.....of water spheres.”
	Eq. (8.24)		this equation should read as:
			$Z_i = (K_w ^2 / K_i ^2)Z_w \quad (8.24)$
	2	6	change to: “..to estimate the equivalent rainfall rate R_s (mm/hr) from the...”
		7	delete “with $Z_w = Z_e$ ”
232	0	10-11	change to: “...a microwave (i.e., $\lambda= 0.84$ cm) path, confirmed...”
234	Eq. (8.30)		right bracket “}” should be matched in size to left bracket “{”
241	0	9	change “...scattering coefficients...” to “...elements of the scattering

- matrix..”
- 242 3 insert after the first sentence: All expectations of the matrix elements are per unit volume (i.e., $\langle s_{ii} s_{jj}^* \rangle$ in Eqs.(8.46) is $n(\mathbf{r}) \langle s_{ii} s_{jj}^* \rangle$). Thus $\langle s_{ii} s_{jj}^* \rangle$ in (8.46) is not the same as that defined in Eq.(8.45).
- 243 0 6 delete “the scattering coefficient”
- 244 2 1 change to read: “...phase shifts of the propagating wave can have..”
- 248 Eq. (8.57) parenthesis “)” needs to be placed to the right of the term “(b/a”
- 249 Eq. (8.58) $\cos^2 \delta$ should be $\sin^2 \delta$; replace k_o with k ; p_v and p_h should be replaced with p_a and p_b respectively
- Eq.8.59a, b change the all the subscripts “h” to “b”, and “v” to “a” in these two equations,
- 2 9 change to read: “ p_a and p_b are the drop’s susceptibility in generating dipole moments along its axis of symmetry and in the plane perpendicular to it respectively, and e its eccentricity,”
- 12-13 rewrite as: “...symmetry axis, and ψ is the apparent canting angle (i.e., the angle between the electric field direction for “vertically” polarized waves, \mathbf{v} in Fig.8.15, and the projection of the axis of symmetry onto the plane of polarization). The forward scattering.....”
- 17 modify to read: “... $f_h = k^2 p_b$, and $f_v = k^2 [(p_a - p_b) \sin^2 \delta + p_b]$ (Oguchi,”
- 3 2 replace “...coefficients...” with “...matrix elements...”
- 4-5 rewrite as: “Hence from Eq.(8.58) an oblate drop has, for an apparent canting angle $\psi = 0$, the following cross sections for h and v polarizations:”
- 268 Fig. 8.29 LDR_{hv} on the ordinate axis should be LDR_{vh}
- 0 1, 4 change LDR_{hv} to LDR_{vh} at the two places it appears in this paragraph.
- 269 Fig. 8.30 in the caption, change LDR_{hv} to LDR_{vh} at the two places it appears.
- 277 0 16 change “23000” to “230,000”

- 289 2 3 delete the sentence beginning with “In this chapter overbars....”
- 298 Fig.9.4a, b here and elsewhere in the text, remove periods in time abbreviations (i.e., should be: “CST”, not C.S.T.)
- 306 2 2 at the end of the sentence on this line, insert: “Radar measurements of wind are biased by the velocity of scatterers (e.g., hydrometeors, insects, etc.) relative to the wind. In this section we consider scatterers are perfect tracers of the wind but later (Section 9.3.3) we introduce corrections for the bias caused by the hydrometeors’ terminal velocity.”
- 390 0 1 change to read “along the path ℓ of the aircraft, and $S_{ij}(K_\ell)$ is the Fourier transform of $R_{ij}(\ell)$. In contrast....”
- 393 1 11 the subscripts on $R_{11}(0)$ should be changed to $R_{ll}(0)$; (i.e., so that it is the same as the subscripts on the second “ D “ in line 19).
- Eq. (10.33) place subscript l on C so that it reads C_l .
- 394 0 1 change to read: “where C_l^2 is a dimensionless parameter with a value of about 2.”
- Eq. (10.37) change to read:
- $$R_{ii}(\rho, \tau_1 = 0) = R(0)[1 - (\rho / \rho_{oi})^{2/3}] \quad (10.37)$$
- 398 1 12 change to read: “...of the weighting function I_n , and $\Phi_v(\mathbf{K})$ is the spatial spectrum of point radial velocities.”
- 17 change to read: “...antenna power pattern under the condition, $\theta_e = \pi/2 - \theta_0 \ll 1$, and....”
- 398 Eq. (10.48) change the subscripts “ ll ” to “ 11 ”.
- 404 2 1 change to read: “...proportional to the radial component of turbulent kinetic energy,....”
- 4 7 place an over bar on the subscript “ u ” in the next to last equation
- 409 3 2 change to read: “...must be interchanged with σ_r , and the second parameter (i.e., $1/2$) in the argument of F must be changed to 2.
- 5, 6 change to read: “Using the series expansion for F to first order in

$1 - \frac{\sigma_\theta^2 r^2}{\sigma_r^2}$, the dissipation rate can be approximated by

- 412 2, 3 2, 1 delete the word “linear” in these two lines.
- 2 5 change “polynomial surface” to “polynomial model”
- 7 change “surface” to “model”
- 419 Fig. 10.18 the “-5/3” dashed line drawn on this figure needs to have a -5/3 slope. Furthermore, remove the negative sign on “s” in the units (i.e., m^3/s^{-2}) on the ordinate scale; this should read (m^3/s^2).
- 445 1 6 delete “time dependence of the”
- 451 Eq. (11.98) change k_z to K_z
- 453 1 10 delete “(s)” from “scatterer(s)”; subscript “c” in $\rho_{c,||}$ should be replaced with subscript “B” to read $\rho_{B,||}$
- 12 a missing subscript on ρ_{\perp} should be subscript “B” so the term reads: $\rho_{B,\perp}$
- Eqs. (11.105 & 106) the symbols $||$ & \perp should also be subscripts, along with “B”, on the symbol “ ρ ” to read “ $\rho_{B,||}$ ” and “ $\rho_{B,\perp}$ ”.
- 454 0 6 change “blob” and “blobs” to “Bragg scatterer” and “Bragg scatterers”
- Fig. 11.11 caption should be changed to read: “....., a receiver, and an elemental scattering volume dV_c .”
- 456 Eq. (11.115) bold “r” in the factor $W(\mathbf{r})$ needs to be unbolded
- Fig. 11.12 add a unit vector \mathbf{a}_o drawn from the origin “O” along the line “ r_0 ”.
- 458 2 4 make a footnote after $\sqrt{2}$ to read: z' is the projection of r' onto the z axis; not to be confused with z' in Fig. 11.12 which is the vertical of the rotated coordinate system used in Section 11.5.4.
- 459 Eq. (11.125) delete the subscript “c” in this equation, as well as that attached to ρ_{ch} in the second line following Eq.(11.125) so that it reads “ ρ_h ”.

- 460 1 4-9 delete the third to fifth sentences in this paragraph and replace with the following:
Condition (11.124) is more restrictive than (11.106); if (11.124) is violated the Fresnel term is required to account for the quadratic phase distribution *across the scattering volume*, whereas (11.106) imposes phase uniformity *across the Bragg scatterer*; this latter condition is more easily satisfied the farther the scatterers are in the far field (also see comments at the end of Section 11.5.3).
- 464 Fig. 11.14 caption: the first citation is incorrect. It should read: “(data are from Röttger et al., 1981)”. Furthermore, delete the last parenthetical expression: “(Reprinted with permission from)”
- 468 2 11 change “(11.109)” to “11.104”
- 475 Eq. (11.151) should be: $\lambda \geq \lambda_y = 10\pi(\nu^3 / \epsilon)^{0.25}$
- 478 0 7 change to read:
“...the gain g . Then g , now the directional gain (Section 3.1.2), is related...”
- 493 1 delete the last sentence and make the following changes:
1) change lines 2 and 3 to read: “... $C_n^2 = 10^{-18} \text{ m}^{-2/3}$ (Fig.11.17), the maximum altitude to which wind can be measured is computed from Eq.(11.152) to be about 4.5 km.”
2) change lines 4 and 5 to read: “...that velocity estimates are made with $\text{SNR} = -19.2 \text{ dB}$ (from Eq.11.153 for $T_s = 3.13 \times 10^{-3} \text{ s}$), and that $\sigma_v = 0.5 \text{ m s}^{-1}$, $\text{SD}(v) = 1 \text{ m s}^{-1}$, and a system temperature is about 200 K (Section 11.6.3).”
- 2 1-4 change to read: “Assuming that velocities could be estimated at SNRs as low as -35dB (May and Strauch, 1989), the WSR-88D could provide profiles of winds with an accuracy of about 1 m s^{-1} within the entire troposphere if C_n^2 values...”
- 8; 9 change “‘14’ to ‘12’; change “able to measure” to “capable of measuring”
- 533 3 change “Hitchfeld” to “Hitschfeld”

SUPPLEMENTS

The following supplements are provided at the indicated places to clarify and/or extend the text of “Doppler Radar and Weather Observations”, Second Edition-1993; 3rd and 4th printings.

Page Para. Line Remarks: Paragraph 0 is any paragraph started on a previous page that carries over to the current page.

xv 4 1 change “meteorology” to “meteorological applications”

List of Symbols

Add:

$E[x]$	Expected value of the random variable ‘x’
\hat{x}	An M sample estimate of $E[x]$,
\bar{x}	Spatial average of ‘x’
K_w	The complex dielectric factor of water; p.35.
k_0	Electromagnetic wavenumber in vacuum
$\langle p \rangle$	A sample average of the property “ p ” of the members

Amend:

	k	Electromagnetic wavenumber ($2\pi / \lambda$) in the atmosphere
3	3	at the end of this paragraph add: R. D. Hill (1990) cited work done by researchers at the Naval Research Laboratory suggesting the first pulse radar detection of aircraft might have been in December 1934.
11	0 6	modify to read: “...earth’s atmosphere (e.g., the speed v of the wave in the atmosphere is slightly slower than its speed c in vacuum).”
18	1 10-11	change to read: “... a is the earth’s radius (approximately 6375 km), and R ...center of the earth to a point on the ray (Fig.2.5).”
19	1 3	amend to read: “... $R= a + h$ (Fig.2.5).”
	Eq.(2.24b)	rewrite as
		$C_0 = \frac{1}{r_c} \approx -\frac{dn}{dh} \quad (2.24b)$
1	16	change to read: “where r_c is the radius of curvature. If n is well...”
Fig.2.6b		Because this figure is also used to derive Equations (2.28) which give the

height h_s and arc distance s of a scatterer S in terms of radar measurements (r, θ_e) , this figure would be more useful if it had an additional ray directed at an elevation angle θ_e . S is located along this ray at an arc distance s . Scatterer S is seen by the radar along an apparent straight line (i.e., the ray path) at an elevation angle θ_e and slant range $s_r = r$. How r is related to the measured apparent range r_a is given in by Eq.(A.7) of Appendix A. The following additional changes to Fig.2.6b are:

(1) change h_e to h_0 to emphasize h_0 is the height of the ray at A when $\theta_e = 0^\circ$; (2) place the characters “R” at the radar location and “A” at the end of the ray RA; (3) extend the radial line a_e to S; (4) label this extension h_s ; (4) label the center of the equivalent earth as O_e ; and (5) show the angle ψ_e which is subtended by radials to R and S from O_e .

Also, to Fig. 2.6a, add the character “R” at the radar location. The caption to Fig.2.6 should read:

Fig.2.6: (a) Circular path of a ray launched at an elevation angle $\theta_e = 0$ from the radar at R on a spherical earth of true radius a . (b) Straight path of the same ray above an equivalent earth of radius a_e . Also shown is the straight path of a ray to a scatterer at S at height h_s and arc distance s observed by radar at a range r and $\theta_e \neq 0$.

- 21 0 7 add the parenthetical entry “(Section 2.2.3.2)” at the end of the sentence.
- 1 1-8 change to read: “Next the radius a_e of the equivalent earth will be related to the refractive index gradient. For our purposes, the solution....to first order in ψ is a good approximation because, for weather radar applications, ranges to precipitation are less than 500 km, and thus $\psi \ll 1$, $\psi_r \ll 1$ (all angles are in radians unless otherwise noted). Substituting this solution into Eq.(2.25).... we find that

$$h \approx \frac{s^2}{2a} - \frac{s^2}{2r_c} \quad ,$$

is the height of the ray launched at $\theta_e = 0$ above the earth of radius a . To obtain the height h_0 of a ray launched at the same $\theta_e = 0^\circ$, to slant range s_r above an equivalent earth of radius a_e (Fig.2.6 b), apply the Pythagorean theorem to triangle O_eRA and relate s_r to $\sin \psi_e$ to obtain the two equations which can be solved giving $h_0 = a_e \left[(\cos \psi_e)^{-1} - 1 \right]$.

Because $\psi_e \ll 1$, and $s = a_e \psi_e$, the approximation

$$h_0 \approx \frac{s^2}{2a_e}$$

is acceptable. Equating h from Fig.26a to h_0 , using the relation $r_c = -(dn/dh)^{-1}$ from Eq.(2.24b), and solving for a_e , we obtain

$$a_e = \frac{a}{1 + a \left(\frac{dn}{dh} \right)} = k_e a \quad (2.27a)$$

where k_e is the equivalent earth radius factor. For the first one or two kilometers of the atmosphere it is customary to set $dn/dh = -1/4a$ (Bean and Dutton, 1966; section 1.5) so in this case the effective radius of the earth is

$$a_e = \frac{4}{3} a \quad (2.27b)$$

13-21 Because Eqs.(2.28b,c) can be derived directly, Eq. (2.28a) is unnecessary.. To demonstrate this derivation, Fig.2.6b needs to be updated as shown in this supplement. Thus delete Eq. (2.28a) and modify the text as:

“.....determine the height and arc distance s to a scatterer at S (Fig.2.6b). In this case all rays launched at θ_e are straight and trigonometric identities can be used to relate the radar measured θ_e and slant range $s_r = r$ to scatter S to obtain the height h_s and arc distance s to the scatterer. Range r is not directly measured, but the range-time delay τ_s of an echo is measured. Range-time is the time delay, after the time a microwave pulse is transmitted, that an echo is received (Section 3.4.2).

The relation for h_s is obtained by applying the law of cosines to the angle at R of the triangle O_eRS (Fig. 2.6b). Then it can be shown

$$h_s = \left[r^2 + (k_e a)^2 + 2rk_e a \sin \theta_e \right]^{1/2} - k_e a . \quad (2.28a)$$

Thus given the range r and elevation angle θ_e of an echo, the scatterer's height can be calculated using (2.28a). The scatterer's arc distance s can be related to r and θ_e by applying the law of sines to angles at R and O_e of the triangle O_eRS and noting that $\psi_e = s / a_e$. Thus

$$s = k_e a \sin^{-1} \left(\frac{r \cos \theta_e}{k_e a + h_s} \right). \quad (2.28b)$$

By substituting for h_s from (2.28a) we have a solution for s in terms of the radar measured parameters r and θ_e . Appendix A gives exact solutions to the arc distance s and height h_s of a scatterer in a spherically stratified atmosphere if the refractive index is any known function of height h .

- 21 delete the last three lines.
- 22 1 4 change to read: "... $\ll 1$ which appears to limit the elevation angle to small values. But shortly it will be shown that the effective earth radius model can be used to higher elevation angles."
- 12-13 change to read: "...given by the Central Radio Propagation Laboratory's exponential reference model $N_s \exp(-h/H)$, one that agrees closely with measured data. Although there are several pairs of N_s and H entries, the pair $N_s = 313$ and $H = 6.95$ km are the climatological average for the United States (Bean and Dutton, 1966, Section 3.8).
- 23 0 At the end of this paragraph add: "Although we use the 4/3rds earth radius model (i.e., $k_e = 1.33$) for this comparison, $k_e = 1.4$ is derived from the exponential model. If we use this k_e for an equivalent earth radius there is less than 300 m height difference (vs. 500 m for $k_e = 1.33$) between the two models at $s = 250$ km. But $k_e = 1.2$ is used by the NWS to determine the height of the WSR-88D's beam above the earth. In this case the height difference between the exponential model and the equivalent earth radius model is 900 m at 250 km.
- 31 Fig. 3.1 There are many definitions for a spherical coordinate system. In this book we attempt to consistently use the one given by the International Organization for Standardization (i.e., ISO) and also used by J. Stratton in his book on Electromagnetic Theory (1941, McGraw-Hill Book Co.), as well as by C. A. Balanis in the third edition of his tome "Antenna Theory" (2005, John Wiley & Sons, Inc.). In this spherical coordinate system, the polar axis is along z (i.e., in the vertical direction), the x axis is Eastward, and the y axis is Northward. The spherical angle θ is the zenith angle (also called the polar angle), and ϕ (or φ) is the azimuth measured counter clockwise (when viewed in the negative z direction) from the x axis. This differs from the azimuth (here designated by "Az") defined by radar meteorologists which is measured clockwise from the y axis—that is, $Az = 90^\circ - \phi$. Moreover, meteorologists consistently use elevation angle,

designated herein as $\theta_e = 90 - \theta$.

32 1 4 strictly power density $S(r, \theta, \phi)$ incident on a scatterer at (r, θ, ϕ) is also a function of beam direction (θ_0, ϕ_0) . For conciseness we do not explicitly show beam direction.

33 1 4 to provide a radiation pattern representative of the WSR-88Ds, change to read:levels, and for the WSR-88D, the normalized power density across the aperture is well approximated by

$$\left[\frac{(1 - 4\rho^2 / D_a^2)^3 + 0.16}{1.16} \right]^2.$$

34 0 1 delete $\left[1 - 4(\rho / D_a)^2 \right]^2$
 2 change the Sherman citation to (Doviak et al., 1998⁴) and replace (3.2a) and the sentence following it with

$$f^2(\theta) = \frac{S(\theta)}{S(0)} = \left[5.405 \left| 1.68 \frac{4! J_4 \left(\frac{\pi D_a \sin \theta}{\lambda} \right)}{\left(\frac{\pi D_a \sin \theta}{\lambda} \right)^4} + 0.16 \frac{J_1 \left(\frac{\pi D_a \sin \theta}{\lambda} \right)}{\frac{\pi D_a \sin \theta}{\lambda}} \right| \right]^2 \quad (3.2a)$$

where $D_a = 8.534$ m, λ is the wavelength (e.g., $\lambda = 0.1109$ m for KOUN a research WSR-88D) and θ is the polar angle measured from the beam axis, not the zenith angle shown in Fig.3.1. KOUN is a research WSR-88D, and J_n is the Bessel function of n^{th} order. This theoretical pattern agrees quite well over most of the angular space except for three ridges of enhanced sidelobes due to three feed support spars. When the beamwidth is small compared to

13 change to read:and the angular diameter of the first null circle (the first null circle is a minimum not a zero) is given by $\theta_{\text{null}} = 4.16 \frac{\lambda}{D}$ (rad). The angular diameter of the first null circle defines the main lobe or beam of the antenna. It has been shown, (3.2b) holds for all WSR-88Ds (Doviak et al., 1998)

⁴ The 1998 report and its errata are available at NSSL's website at www.nssl.noaa.gov under publications/reports.

- 2 6 insert; If the shape of the radiation pattern of a beam, not necessarily circularly symmetric, is well approximated by the product of two Gaussian functions, the maximum directivity is
- $$g'_i = \frac{1}{\sigma_\phi \sigma_\theta},$$
- where σ_ϕ^2 and σ_θ^2 , assumed to be much smaller than 1 rad^2 , are the second central moments of the two-way pattern expressed in normal form. The two-way pattern is the product of the transmitted radiation pattern and the receiving antenna's field of view pattern. Typically the same antenna is used for both transmitting and receiving, and for circularly symmetric Gaussian shaped pattern, the two-way pattern is $f^4(\theta) = \exp\{-\theta^2 / 2\sigma_\theta^2\}$. In terms of the one-way 3-dB pattern width θ_1 , $\sigma_\theta = \theta_1 / 4\sqrt{\ln 2}$ (for more discussion see Section 5.3).
- 35 0 1-2 at the end of this sentence add: Strictly the power gain pattern is the pattern of gain in power **density**.
- 1 There are several definitions of cross sections. For example,
 $\sigma_d(\theta', \phi') = \frac{S_r}{S_i} r^2$ is the *differential scattering cross section*; that is, it is the cross section *per unit* solid angle. Integration of $\sigma_d(\theta', \phi')$ over 4π steradians gives the *total scattering cross section* σ_s (Section 3.3).
- 4 6 K_w is used quite often but has not been given a name, it is recommended to call it the “complex dielectric factor of water”. Thus rewrite this line as: “where K_w , the complex dielectric factor of water, = $(m^2 + \dots)$ ”
- 36 0 2 insert at the end of the first sentence: “It can be shown, using formulas presented in Section 8.5.2.4, that Eq. (3.6) has practical validity only if drops have an equivalent spherical diameter D_e less than 2 mm. Drops having D_e larger than 2 mm have backscatter cross sections differences larger than about 0.5 dB for horizontally and vertically polarized waves (i.e., $\sigma_{bh} > 1.1\sigma_{bv}$).”
- 38 1 4 add at the end of this paragraph: “Furthermore, Probert-Jones (1984) demonstrated that internal resonances in electrically large low-loss spheres can generate greatly enhanced scatter in both the forward and backward directions.
- 2 1 change to “...by water or ice hydrometeors, radars with shorter....”

- 2-4 to be consistent, change “drop(s)” and “particle(s)” to “hydrometeor(s)” everywhere in these paragraphs.
- 5 change to read: Assume S_i incident on hydrometeors within an elemental volume $\Delta V(r)$ does not significantly change across this volume. That is, the field in $\Delta V(r)$ is assumed to be the same whether hydrometeors are present within $\Delta V(r)$ or not—nevertheless hydrometeors along the path between the radar and $\Delta V(r)$ can significantly alter S_i as shown in Section 8.5.2.3. This assumption is referred to as the Born (or single-scattering) approximation, In this case the power density change ΔS_i in a.....”
- 39 0 7 change to “....density $S_i(r_2)$ at range r_2 given $S_i(r_1)$ is,”
- 13-14 change to “....coefficient, and $\sigma_e \equiv \sigma_a + \sigma_s$ is the extinction cross section. $N(D, r)$, named the hydrometeor size distribution, is the expected number density of hydrometeors per unit diameter having the extinction cross section σ_e . Specific attenuation is a function of r both because hydrometeor density can change from location to location as suggested by Eq.(3.11b), and because hydrometeor structure (i.e., liquid or ice water and its shape), can change with r (i.e., σ_e can also be function of r). Consequently the specific attenuation k is, in general, a function of r . The product $N(D, r)dD$ gives...”
- 1 5 change to read as: “...where l is the one-.....loss factor due to...”
- 42 Fig. 3.5 caption: Because there is considerable confusion concerning the use of the unit dBZ, and because some writers use dBz for the decibel unit of reflectivity factor Z , we present the following comment:

The logarithm decibel dB is not an SI unit. But dB has been accepted widely as a “unit” (e.g., Reference Data for Radio Engineers, 5th Edition, Howard W. Sams, publisher, division of ITT, p.3-3). The decibel is also recognized by international bodies such as the International Electrotechnical Commission (IEC). The IEC permits the use of the decibel with field quantities as well as power and this recommendation is followed by many national standards bodies. Moreover, according to SI rules, units should not be modified by the attachment of a qualifier. Nevertheless, appendages to dB have been *accepted in the engineering field* to refer the dB unit to a reference level of the quantity being measured. For example, dBm is the decibel unit of power and is equal to $10 \log_{10} P/P_r$ where P is the power, in milliwatts, referenced to $P_r = 1$ milliwatt (e.g., Reference Data for Radio Engineers). Similarly, the parameter dBZ *has been accepted by the AMS* as the symbol for the “unit” decibel of reflectivity factor Z referenced to reflectivity factor $Z = 1 \text{ mm}^6\text{m}^{-3}$ (Glossary of Meteorology, 2nd Edition, 2000, American

Meteorological Society).

43 3 6 for clarity of units revise to read: ‘...therefore specific attenuation (m^{-1})..’

44 3 4 Blake has more recently published (1986, in “Radar range performance analysis”, 2nd ed., ARTECH House, Norwood, MA.) new values of attenuation in gases. For example, at $\lambda = 10$ cm, $r = 200$ km, $\theta_c = 0^\circ$, the two way loss is about 0.3 dB larger than that given in Fig.3.6.

47 Table 3.1 1) The entry for the 6 dB bandwidth B_6 for the short pulse “Filter” is not matched to the short pulse . The matched B_6 for the short pulse is $1.04/\tau$ (Eq. 3.39) = 0.66 MHz.
2) The reflector diameter is 8.534 m (28 feet) and has a focal length to diameter ratio $f/D = 0.375$.

48 2 8 for clarification change to read: “...STALO connected to the detector; in this case the detector—only one is needed---is a video detector and its output is the envelope of the rf echoes) when...”

49 In Fig.3.7 and its associated text we introduce the term ‘range time’, but elsewhere we connect ‘range’ and ‘time’ by a hyphen (e.g., Fig.4.1). To be consistent, and because range and time in the context of these figures are different representations of one and the same variable, it is suggested that everywhere it applies, we should use ‘range-time’ (ditto for ‘sample-time’).

56 Eq. (3.34) if the beam is passing through clouds and storms, Eq. (3.34) should be replaced by

$$T'_s = \left(1 - \frac{1}{\ell_c}\right) (1 - \chi + \chi\eta_r) T_c + \frac{1 - \chi}{\ell_c} T_s + \chi(1 - \eta_r) T_g + \frac{\chi\eta_r}{\ell_c} T_s$$

where ℓ_c and T_c are the cloud’s attenuation and temperature.

58 1 4 change to read: “...effects is reached (Section 4.5).”

57 Fig. 3.11 for completeness, the ordinate should be labeled “Sky noise temperature T_s (K)”

60 1 2 change to: “...noise power N . This peak signal-to-noise ratio, $S(0)/N$, readily obtained.....

(3.38) change SNR to $S(0)/N$ in this equation and on line 5.

7 change to: "...that the signal-to-noise ratio SNR is maximized..."

64-121 because Chapters 4 and 5 focus attention mainly on weather signals, changing power P to signal power S throughout these chapters should improve clarity. This suggestion is made because P is generally used to define signal S plus noise N power.

Fig. 4.2 caption change last line to read: "...at τ_{s1} (Section 4.4.2). This figure assumes the transmitted pulse is initiated at exactly the same time as the timing pulse; more commonly there is a transmitted pulse delay τ_T between the timing and transmitted pulses (see revised figures 4.5, and 4.6).

61 Eq. (3.40b) elsewhere in the book (e.g., pp.172, 173), we define v_a as the Nyquist velocity. It would be less confusing if v_a is replaced with v_N throughout the book; this change keeps one from calling v_a the unambiguous velocity which is confusing because all velocities in the interval $-v_N < v < v_N$ are unambiguously measured.

68 Eq.(4.2) express this equation as:

$$\begin{aligned} P(\tau_s) &= \alpha VV^* = \alpha \frac{1}{2} \sum \dots\dots \\ &= \alpha \frac{1}{2} \sum \dots\dots \end{aligned}$$

where α proportionality constant.

68 3 8 add: "In most of this book" the expected value $E[x]$ is assumed to be obtained by averaging over an ensemble of scatterer configurations ξ having the same statistical properties (e.g., reflectivity factor, velocity field, etc.). But the expectation operator $E[x]$ is used for averages of ensembles of other variables (e.g., , scatterer cross sections in Section 4.4). To avoid subsequent possible confusion, we could have appended the subscript ξ to have $E_\xi[x]$, but the subscript ξ will be used only where there is a need to avoid confusion."

71 2, 3 an explanation for the $\sqrt{2}$ factors that the errata stated needs to be inserted into Eqs. (4.4) and (4.6) might be helpful. Because a lossless receiver is assumed, the sum of powers in the I and Q channels must equal the power at the input to the receiver (i.e., the synchronous detectors in Fig. 3.1). Because we have assumed the amplitude of the echo voltage at the receiver's input is A (e.g., Eq. (2.2b)), the amplitude of the signal in the I and Q channels must be $A/\sqrt{2}$. Furthermore, we can determine from Eq.

(4.5) that the rms values of I and Q voltages equals σ (i.e., $I_{\text{rms}} = Q_{\text{rms}} = \sigma$). Thus the average power in each of the channels is σ^2 , and the sum of the average powers in these two channels is $2\sigma^2$ which equals the expected power $E[P]$ at the input to the receivers. The constants of proportionality (i.e., impedances) that relate voltage to power are assumed the same at all points in the receiver (e.g., at inputs to the I and Q channels).

72 2 2

here and in Eq. (4.8), we need to distinguish the expectation operator “ $E[x]$ ” used in Sections 4.2 and 4.3, which implicitly applies to an ensemble of scatterer configurations, from the expectation over the scatterer’s cross section given by Eq.(4.8). To emphasize this difference, we rewrite Eq.(4.8) and revise the text as:

“...and the expected backscattering cross section σ_b . The expected power contribution $E_\sigma [P_i]$ from each scatterer to $E[P(\tau_s)]$ is,

$$E_\sigma [P_i] = \frac{1}{2} \alpha |W_i|^2 E_\sigma [|A_i|^2] \quad (4.8)$$

where $\alpha E_\sigma |W_i|^2 [|A_i|^2] / 2$ is the i^{th} scatterer’s expected echo power which can be directly.....using Eq.(3.24). The subscript ‘ σ ’ is appended to ‘ E ’ to differentiate this expectation from $E_\xi[x]$ which is an expectation made over the configurations ξ of scatterer locations (Supplement 68, 3, 8).

Sample time averages provide estimates of both expectations, because hydrometeors change their cross sections (due to wobble and oscillations) simultaneously and they change their relative locations. Thus, henceforth the expectation ‘ E ’ without subscript denotes the combined average over all variables that cause the echo voltage to fluctuate. An average over the ensemble of backscattering cross sections is needed because a hydrometeor undergoes continuous.....as it falls through the air. Thus its backscattering fluctuates about an expected value. Therefore the expected value σ_b (i.e., ...is needed to determine $E[P(\tau_s)]$ ”

Eq.(4.9a)

To explicitly differentiate between expectation ‘ E ’ over all random variables that cause weather signal fluctuations, and the expectation of the hydrometeors’ cross sections, rewrite this equation as:

$$E[dP] = \frac{1}{2} \alpha \sum_i |W_i|^2 E_\sigma [|A_i|^2] \quad (4.9a)$$

73 2 3

To account for drops not being spherical change to:
“.....per unit volume. In this derivation it has been assumed that drops are spherical, but in fact they are oblate (Fig.8.1). Thus the backscattering

cross section depends on the polarization of the wave. If waves are horizontally or vertically polarized, the backscattering cross section is noted as either σ_{hh} or σ_{vv} and reflectivity as either η_h or η_v (more detailed discussion is given in the supplements to pp. 240-242).

74 1 4 change to read: "...to its range extent, and if the polar axis of the spherical coordinate system (Fig. 3.1) is aligned with the beam axis (i.e., $\theta_0 = 0$), we can approximate Eq. (4.11) by..."

8 change to read: "...by a Gaussian shape, $f^4(\theta) = \exp(-\theta^2 / 2\sigma_\theta^2)$, we can show..."

74 Eq. (4.16) to be have left and right sides of this equation consistent, write reflectivity and loss factor as $\eta(\mathbf{r}_0)$ and $\ell^2(\mathbf{r}_0)$

10 change to read: "where $\theta_1 = 4\sigma_\theta \sqrt{\ln 2} \ll 1$ is the of the one-way power pattern."

76 0 1 change to read: "...is, assuming a shell of thickness $dr \ll \lambda$, scatterers in that shell return an echo voltage"

4 insert the sentence: Eq. (4.17) assumes $dr \ll \lambda$ and scatterers do not introduce any phase shift upon backscatter (i.e., scatterers have negligible phase shift).

Eq. (4.18a) the text would read better if Eq. (4.18a) and the sentence containing this equation were moved to follow the first sentence in paragraph 1.

1 6 to have consistency between the power response $|G(f)|^2$ and its 6 dB bandwidth B_6 (a parameter commonly used to define a "matched condition") change this line to read: "...is rectangular and $|G(f)|^2$ has a Gaussian shape with a 6 dB bandwidth, B_6 equal to"

1 12-15 because the comment about causality is not consistent with Fig. 4.5, and to include the time delay τ_T in arrival of the transmitter pulse at the antenna aperture relative to the start of the timing pulse (Fig. 4.2) and the time delay τ_R of the received echo propagating from the antenna aperture to the receiver output, Fig. 4.5 should be modified as follows:

Insert the symbol τ_R between the end of the function $h(\tau_s - t)$ and the tick mark at τ_s . To designate the start of the timing pulse, place another vertical line to the left of the existing one (this designates the start of the

transmitted pulse) and insert the symbol τ_T between it and the existing line. Third, extend the horizontal line marking τ_s to the new vertical line. Finally insert the symbol τ_p between the point at which $h(\tau_s - t)$ rises on its right side and a vertical line through the peak of $h(\tau_s - t)$.

Furthermore, lines 15, 16 should read as: "...are obtained (Fig. 4.5), accounting for: 1) the transmission line delay τ_R of the echo's arrival at the antenna and its appearance at the output of the receiver, and 2) the time difference τ_T between the start of the timing pulse and the arrival of the transmitted pulse at the aperture (Fig. 4.5), and substituting $t' = \dots$ "

After Eq. (4.18b) write: In Fig.4.5, τ_R is the time required for the received echo to propagate from the antenna aperture to the receiver output, τ_p is the time, after arrival of the scatterer's echo at the receiver's output, for the echo to reach a maximum amplitude, and τ_T is the time at which the transmitted pulse arrives at the antenna aperture after the start of the timing pulse (note that τ_T can be positive or negative).

- | | | | |
|----|---|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 77 | 0 | 5 | insert after Eq.(4.19): In writing Eq. (4.19), time delays τ_T and τ_R are assumed to be zero; otherwise the upper limit should read as $c(\tau_s - \tau_T - \tau_R) / 2$. |
| 78 | 0 | 2 | for clarity, here and everywhere in this section including Fig. 4.6, change τ_t to τ_R . |
| | | Fig. 4.6 | insert the symbol τ_p to designate the time between the arrival of the echo at the receiver's output to the time the echo is a maximum. |
| | 0 | 4-5 | change to read: "...and if transmitter delay τ_T is zero, would..." |
| | 1 | 3 | change to read: "...delay $\tau_t = \tau_T + \tau_R + \tau_p$ is the sum of the" |
| | | 10 | change to read: "estimate the range to scatterers that are contributing most to the echo sample (this assumes scatterers are all of the same cross section). That is, a voltage sample...." |
| 79 | 0 | 8 | add at the end of this paragraph: Range r_0 is the range to the center of the radar's resolution volume defined in Section 4.4.4. |
| 80 | 2 | 2-3 | change to read: "...The 6 dB angular width θ_6 of $f^4(\theta, \varphi)$... function) is taken to be the angular width of V_6 because it equals θ_1 the 3 dB width of |

the one-way power pattern, a commonly accepted definition of angular resolution.

81 0 11 change to read: "...width $r\theta_1 = r\theta_6$, and..."

82 1 Eqs. (4.31) and (4.32) without the subscript "w" apply to all spherical particles, not only water. Thus to emphasize this point, and to make these two equations consistent, modify line 3 to read:

"If scatterers are spherical..."

Then delete the subscript 'w' in (4.31) and (4.32) and rewrite (4.31) and the line that follows as:

$$\eta_m(\mathbf{r}) = \frac{\pi^5}{\lambda^4} |K_m|^2 Z(\mathbf{r}), \quad (4.31)$$

where K_m is the dielectric factor of the medium filling the sphere and

(4.32)

is the reflectivity factor of spheres. For water spheres the subscript 'w' is appended to K and Z . Whenever the Rayleigh....

83 Sections 4.2 Change heading to "The Per-Pulse Signal-to-....." and the first line to "The per-pulse SNR for distributed....."

2 For further insight and interpretation into the discussion in this paragraph add: "This result can be looked upon as being similar to a generalized uncertainty principle. Here it is applied to the realization that we cannot simultaneously concentrate the signal $f(t)$ in the time domain and its spectrum $S(f)$ in the frequency domain. A measure of concentration are the second central moments σ_f^2 and σ_t^2 , and these two measures satisfy the condition

$$\sigma_f^2 \times \sigma_t^2 \geq \frac{1}{4\pi}.$$

Equality is achieved only for the Gaussian pulse (Harris, 1978⁵)."

89 3 6-8 To extend the discussion to include M being odd rewrite these lines as follows: delete '(M/2)' on line 6, and rewrite lines 7 and 8 as:
 "...done in order to have an equal number of positive and negative Doppler lines to the right and left of the zero ($k = 0$) line. If M is even, the power $|Z(Mf_0/2)|^2$ is split so there..."

⁵ Harris, F. J., 1978: On the use of Windows for Harmonic Analysis with the Discrete Fourier Transform. Proc. IEEE, Vol. 66, No.1, January.

- 92 3 1 To alert the reader of the transition to weather signals, insert the following at the beginning of Section 5.1.2 and modify the first sentence as shown:
 Voltage V in the preceding sections is not necessarily a random variable as is the weather signal. In this and the following sections, the focus will be on random voltages. The Fourier transform.....and correlation of sample sequences of the random voltage generated by weather echoes. The output
- 94 2 1 statistical properties of a voltage sample sequence are, for example, its expected mean square value (i.e., proportional to expected power), its autocorrelation function, etc.
- 95 0 this paragraph should be clearer, and better connected to the properties of the scatterers, by revising it to read from the 4th to the 10th lines as:
 “...the same environmental conditions (e.g., the same wind field). Although the environmental conditions might not change, the configurations of the scatterers during another observational period can be altogether different, and the changing scatterer configuration will produce another sample sequence both sharing the same statistical properties (e.g., the reflectivity). The statistical properties of weather signals can be estimated reasonably well from one member of the ensemble of all possible sample sequences if the single sample sequence is of sufficiently long time duration (for weather radars, this duration is typically >0.05 s). If the statistical properties of the ensemble can be deduced from samples from one member of the ensemble of sequences, the samples $V(m)$ are said to be ergodic. Only then can the properties of an ensemble be inferred from processing $V(m)$ s. Sample sequences produced by scatterers with statistically stationary properties are members of an ergodic ensemble. We shall assume.....”
- Eq. (5.17) note that this equation is a biased estimate of the autocorrelation $R(l)$. To obtain an unbiased estimator, replace the M^{-1} multiplier with $(M-1-|l|)^{-1}$.
- 106-112 because Sections 5.2 and 5.2.1 stipulate that the velocity field is steady (i.e., time independent), and to be consistent with later notation (i.e., errata on p. 113, line 1), we could have, in these two sections, appended the subscript ‘s’ to ‘ v ’ to emphasize that ‘ v ’ is a steady wind.
- 110 Eq. (5.50) $\sigma_v^2(\mathbf{r}_0)$ is sometimes defined as the variance of the mean normalized Doppler spectrum. In that case the Doppler spectrum is interpreted as showing the probability density of velocities. This is at best a questionable interpretation, because Doppler velocities are a composite of real and apparent velocities (Fang and Doviak, JAOT, Dec. 2008, pp. 2245-2258).

- 112 2 10-12 change to read: "...turbulence, etc.) act independently as we now demonstrate."
- 113 footnote 5 Add to the footnote: $|F_i(0)|^2$ is only the weight impressed on the i^{th} scatterer if the antenna is not scanning. Because the antenna pattern on transmission is in a different direction on reception, for azimuthally scanning antennas, $F_i(0) \propto f(\phi_0, \phi_i) f(\phi_0 + \alpha\tau_s, \phi_i)$.
- 114 0 8 change to read: "...where σ_{bk} is the expected backscattering cross section of the k^{th} hydrometeor (expectations computed over the ensemble of cross sections for the k^{th} scatterer), it follows ...
- 116 at the end of Section 5.2, add the following paragraph:
In this section we assumed scatterers follow exactly the air motion. But usually scatterers are hydrometeors that fall in air, have different fall speeds because of their different sizes, and change orientation, and vibrate (if they are liquid). These hydrometeor characteristics broaden the Doppler spectrum associated with the velocity field increasing $\sigma_v^2(\vec{r}_o)$ obtained from Eq. (5.51).
- 2 6-8 Change to read: "...of scatterers, shear of v_s can causeas can differences in the terminal velocities of various...."
- 117 1 1 Change to "...components of terminal velocities of the assorted-size..."
- 118 0 after Eq. (5.75): It should be noted that as $\theta_0 \rightarrow 0$, the angular shears in Eq. (5.74) should be replaced by k_θ along the two principal axes of the beam pattern. For example, if the beam is circular symmetric and $\theta_0 = 0$, $\sigma_s^2 = r_o^2 \sigma_\theta^2 [k_\theta^2(\phi = 0) + k_\theta^2(\phi = \pi/2)] + (\sigma_r k_r)^2$.
- after Eq. (5.76): It should be noted that if the receiver bandwidth B_6 is much larger than the reciprocal of the pulse width τ , $\sigma_r^2 = \frac{1}{12} \left(\frac{c\tau}{2} \right)^2$.
- 124-131 to be consistent with notation used in Chapter 5 and that used after page 131, we should change 'k' to 'm' and 'm' to 'l' wherever it appears in these seven pages.
- 125 2 1 change to read: "It was explained in Section 5.2.2 that weather...."
- 5 change to read: "...assume a Gaussian signal power spectrum plus white noise,

Eq. (6.3) because $S(\nu)$ is the signal plus noise power spectrum, and because P generally defines signal S plus N power (e.g., 3rd line from bottom of this page), it is suggested $S(\nu)$ be replaced with $P(\nu)$.

Eq. (6.5) add subscript ‘s’ to ‘ ρ ’ to distinguish the correlation coefficient of the weather signal from the correlation coefficient ρ_{s+n} of weather signal plus noise to be introduced by Eq. (6.13d) of this supplement.

128 0 2 for clarity, replace ρ_s , here and elsewhere, with $\rho_s^{(sq)}$.

2 4 to place more specificity on the values of M and σ_{vn} for which the equation $M_I = 2M\sigma_{vn}\pi^{1/2}$ is valid, it can be shown that the condition $2.4/M \leq \sigma_{vn} \leq 0.25$ applies, or in terms of σ_v ,

$$4.8v_N / M \leq \sigma_v \leq v_N / 2,$$

where v_N is the Nyquist velocity (i.e., $\lambda / 4T_s$). Thus the larger is M , the smaller is the allowed spectrum width to use the simplified expression $M_I = 2M\sigma_{vn}\pi^{1/2}$. Nevertheless, this range of spectrum widths covers most conditions observed with 10 cm wavelength weather radars.

Eq.(6.13): this equation is valid when signal power is much stronger than noise power. Furthermore, in the fourth line of the paragraph leading to this equation, the SD of \hat{S} is only proportional to $\sqrt{P^2+N^2}$ if the number of independent samples of P and N are equal; in general this is not the case. For example, noise power in the WSR-88D is estimated independently of the signal power, and its estimate uses many more samples than that used to estimate signal plus noise power. Therefore, the following text, replacing paragraph 3 on p.128, gives the standard deviation of Z (dBZ) estimates as a function of Signal-to-Noise ratio assuming noise is estimated without significant variance.

To estimate reflectivity factor Z in presence of receiver noise, receiver noise power N needs to be subtracted from the signal plus noise power estimate \hat{P} (i.e., $P = S + N$). Thus the Z estimate is $\hat{Z} = \alpha\hat{S} = \alpha(\hat{P} - N)$ where \hat{P} is a uniformly weighted M sample average estimate of the power P at the output of the square law receiver (as in the WSR-88D), and α is a constant calculated from the radar equation. N is usually measured during calibration but can also be estimated in parallel with weather data collection (Ivić et al. 2013)⁶. In both cases, many more samples are used to obtain its estimate. Therefore its variance is negligibly small compared to

6 Ivić I. R., C. Curtis, and S. M. Torres, 2013: Radial-based Noise Power Estimation for Weather Radars, *J. Atmos. Oceanic Technol.*, **30**, 2737-2753. <http://journals.ametsoc.org/doi/abs/10.1175/JTECH-D-13-00008.1>

that of \hat{P} , and the noise power N is nearly its expected value⁷. Z is usually expressed in decibel units; that is, $\hat{Z}(dBZ) = 10\log_{10} \hat{Z} = 10\log_{10}(\alpha\hat{S})$ where \hat{Z} units are $\text{mm}^6 \text{m}^{-3}$. The error in decibel units is now derived.

Let \hat{S} , the M sample estimate of signal power, be expressed as $\hat{S} = S + \delta S$ where δS is the displacement of \hat{S} from S . Thus

$$\hat{Z}(dBZ) = 10\log_{10}(\alpha S) + 10\log_{10}\left(1 + \frac{\delta S}{S}\right) = Z(dBZ) + \delta Z(dBZ) \quad (6.13a)$$

Because $\hat{S} = \hat{P} - N$ where N is a constant, $\text{var}[\hat{S}] = \text{var}[\hat{P}] = P^2 / M_1$ (i.e., Eq.6.10). If the equivalent number of independent samples M_1 is sufficiently large so that $\delta S / S$ is small compared to 1, the second \log_{10} term in Eq. (6.13a) can be expanded in a Taylor series. Retaining the dominant term of the series, the estimated reflectivity is well approximated by

$$\hat{Z}(dBZ) \approx Z(dBZ) + 4.34 \left(\frac{\hat{S}}{S} - 1 \right). \quad (6.13b)$$

The first term and the constant 4.34 are not random; thus the standard error in the estimate $\hat{Z}(dBZ)$ is simply $SD[\hat{Z}(dBZ)] = 4.34 SD[\hat{S} / S]$. Because the signal power estimate \hat{S} is derived by subtracting the noise power N , a fixed number, from the measured power estimate \hat{P} , the standard deviation of \hat{S} is $SD[\hat{S}] = SD[\hat{P}] = P / \sqrt{M_1} = (S + N) / \sqrt{M_1}$ where M_1 is the number of independent signal plus noise power samples.

$$SD[\hat{Z}(dBZ)] = \frac{4.34 \left(1 + \frac{N}{S}\right)}{\sqrt{M_1}} \text{ (dBZ)}. \quad (6.13c)$$

The M_1 contained in the M sample set, can be calculated from (6.12) in which $\rho^{(sq)}(mT_s)$ is replaced by $\rho_{s+n}^{(sq)}(mT_s)$ the magnitude of the correlation coefficient of the signal plus noise samples, which is calculated in the following paragraph.

The correlation of the input signal plus noise (Eq. 6.4) is normalized by $S + N$ to obtain the correlation *coefficient* of the input signal plus noise power estimates. Because $\rho_{s+n}^{(sq)}(mT_s)$ is the square of ρ_{s+n} , the correlation coefficient of the input power samples is,

⁷ Although N can be calculated with negligible variance, the expected value can depend, especially for high performance radars such as the WSR-88D, on beam direction due to external noise sources factors such as the sun, earth temperature, precipitation, etc.

$$\rho_{s+n}^{(sq)}(mT_s) = \left(\frac{S}{S+N} \exp\{-2(\sigma_{vn}\pi m)^2\} + \frac{N}{S+N} \delta_m \right)^2. \quad (6.13d)$$

Then $\rho_{s+n}^{(sq)}$ can be substituted into (6.12) in place of $\rho_s^{(sq)}$, and the sum numerically evaluated to obtain M_I . If this result is substituted into (6.13c) we obtain a solution for the standard error (in dBZ units) of the reflectivity factor as a function of the number of samples M , the normalized width, σ_{vn} of the weather signal's Doppler spectrum, and the signal to noise ratio S/N .

Alternative to a numerical solution, there are conditions whereby we can obtain a useful analytical solution. It can be shown the condition $\sigma_{vn} \leq 0.25$ still applies to replace the sum in Eq. (6.12) by an integral even if $N \neq 0$; furthermore, $\sigma_{vn} \geq 2.4/M$ becomes a sufficient condition to ignore the term $|m|/M$ in that equation. If the dwell time MT_s is much longer than the correlation time of the weather signal samples, $\rho_{s+n}(MT_s) \ll 1$ and thus the limits on the integral can be extended to infinity. Evaluation of this integral under these conditions yields

$$M_I = \frac{\left(1 + \frac{S}{N}\right)^2 M}{1 + 2\frac{S}{N} + \frac{(S/N)^2}{2\sigma_{vn}\sqrt{\pi}}} \quad (6.13e)$$

The formula for calculating the standard error in estimating $Z(\text{dBZ})$ as a function of S/N is obtained by substituting (6.13e) into (6.13c) yielding

$$SD[\hat{Z}(\text{dBZ})] = \frac{4.34}{\sqrt{M}} \sqrt{\left(\frac{N}{S}\right)^2 + 2\frac{N}{S} + \frac{1}{2\sigma_{vn}\sqrt{\pi}}} \quad (\text{dB}), \quad (6.13f)$$

a result agreeing with that obtained from simulation (Melnikov, 2011)⁸. Although $SD[\hat{Z}(\text{dBZ})]$ increases without bound as S/N decreases, it can be shown SD of \hat{Z} estimates, expressed in linear units (e.g., $\text{mm}^6 \text{m}^{-3}$), decreases as S/N decreases and reaches a minimum value at $S = 0$ (i.e., $SD[\hat{Z}(\text{mm}^6 \text{m}^{-3})] = \alpha N / \sqrt{M}$ if $S = 0$; in this case $\hat{Z} = 0$)! This apparent dichotomy is explained by the fact that the slope of the logarithmic function increases without bound as its argument decreases. That is, a constant width of a distribution on the linear scale increases on the logarithmic scale as the mean decreases.

133 Eq.(6.20a) A slightly more rigorous condition to replace Eq.(6.20a), one based upon the correlation function for the linear receiver (condition 6.20a was derived based on power estimates using a square law receiver) and estimation of the first and second moments, is: $\sqrt{2\pi}M\sigma_{vn} \gg 1$.

⁸ Melnikov, M. et al., 2011: "Mapping Bragg Scatter with a Polarimetric WSR-88D", *Jo. Atmos. & Oceanic Tech.*, **28**, Oct. 1273-1285.

- 136 4 At the end of this paragraph add: Furthermore, Eq.(6.29) gives the spectrum width for any shape spectrum provided T_s is much smaller than the signal's correlation time. This is so because the second central moment of the Doppler spectrum is the derivative of the autocorrelation function at zero lag, and because under the stipulated condition the derivative is well approximated by the finite difference formula of Eq. (6.29).
- Figs. 6.6 and 6.7 Because S/N is a ratio of numbers, and because when ratios are expressed in dB, the nomenclature SNR is used, S/N in these figures should be replaced with SNR.
- 150 To distinguish k_h and k_v , the **incremental wavenumbers**, from the free space wavenumbers k_h and k_v , the free space wavenumbers of the H and V polarized waves which are vastly different in magnitude, change k_h and k_v , to k'_h and k'_v .
- 1 8 to remove ambiguity, insert after the end of the sentence: Here ϕ_{DP} is the total differential phase shift but later (i.e., Section 8.5.2.3 and thereafter) ϕ_{DP} is the differential phase shift associated only with propagation. But, for Rayleigh scatter $\phi_{DS} \approx 0$, so the ambiguity typically is not significant.
- 195 2 1-3 change to read: "Three parameters of interest..... integration, (2) the effective beamwidth, and (3) the effective pattern shape."
- 195-196 to be consistent with notation used elsewhere, and because on these pages we exclusively use the term "effective" instead of "apparent" when we describe patterns broadened by azimuthal scanning, use the subscript "e" instead of "a" on $f_a(\phi - \phi_0)$ and ϕ_a .
- 196 1 1&7 replace ϕ_a with θ_{1e} .
- 1 at the end of this paragraph add the sentence: "If $\alpha MT_s \leq \theta_1$, the effective normalized pattern retains a Gaussian shape, but if $\alpha MT_s \gg \theta_1$, the pattern shape is roughly trapezoidal having amplitude $(\alpha MT_s)^{-1}$, and a one-way half-power width of about αMT_s ."
- 2 3&5 replace ϕ_a here and in Fig. (7.25) with θ_{1e} .
- Fig. 7.28 Specified not-to-exceed sidelobe levels given by the dashed lines in Fig.7.28 applies to the WSR-88D radiation pattern **with radome**. But patterns without radome are specified as follows: from -26 dB at $\pm 2^\circ$ to -38 dB at $\pm 10^\circ$, and then the constant level should be at -42 dB. The

dashed lines in this figure principally limit the ridge of enhanced sidelobes generated by the feed support spars.

The pattern plotted is one taken without radome and is one along the vertical plane which does not show increase in sidelobe levels due to spar blockage. Thus the measured sidelobe levels are considerably below the specified not-to-exceed levels. Spar blockage increases sidelobes by about 5 dB at 2° to about seven dB at 10°. However these feed support sidelobes only occupy a small fraction of the entire sphere of radiation, and thus contribute less unwanted echo power than if the entire angular sector of the reflectivity field outside the mainlobe was illuminated by sidelobe levels meeting the specifications. The envelopes of measured one-way radiation patterns beyond ± 20° and theoretical pattern calculations⁹ suggest far out sidelobes of the WSR-88D are 55 dB below the peak of the mainlobe for most of the angular space.

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| 203 | 1 | 4 | Rewrite as “equal to or larger than the reciprocal...” |
| 204 | 0 | 7 | Change to read “...wires, etc. The clutter spectrum width” |
| 212 | 0 | | add at the end of this paragraph: More recently Clothiaux et al. (JTECH, 1995) present cloud reflectivities much weaker than those reported by Gossard and Strauch (1983). Clothiaux et al. calculated cloud reflectivity factors using published in situ measurements of mean droplet diameter and liquid water content, and assumed a gamma drop diameter distribution with a shape factor of 18 (i.e., the distribution is nearly normal). Clothiaux et al., found that continental cumulus has Z values ranging from -28 to -15 dBZ (3 cases); -35 to -38 dBZ for continental stratocumulus (2 cases); -35 to -15 dBZ for marine stratocumulus (22 cases); and the weakest reflectivity -52 to -30 dBZ is associated with altocumulus clouds (5 cases). |
| 234 | 2 | | remarks: |

The fitted exponent $b = 4.24$ for the power law fit to $f_h - f_v = aD_e^b$ was obtained using the Gans (1912) approximation for the forward scattering coefficients f_h , f_v of oblate drops. Although the calculations were not explicitly shown, the Gans approximation (often called the Rayleigh-Gans theory) is explicitly shown for the computation of the backscattering matrix coefficients presented in Section 6.5.2.4. The Gans approximation is crude compared to more precise computation made using the T matrix¹⁰. The T matrix computations indicate the best

⁹ Errata for report “Polarimetric Upgrades to Improve Rainfall Measurements” National Severe Storms Laboratory Report, April, 1998 can be found on NSSL’s website under “publications” and then “other”.

¹⁰ Mishchenko, M.I., 2000: Calculation of the amplitude matrix for a non-spherical particle in a fixed orientation. *Appl. Opt.*, **39**, 1026-1031.

exponent is $b = 4.8$ (at the frequency of 2.705 GHz). Thus, using about 47,000 DSD measured with an 2D video disdrometer in Oklahoma and the more accurate exponent b , the following better relation between rain rate and K_{DP} is

$$R = 44 (2.705 K_{DP}/f)^{0.822}$$

where f is the radar frequency in GHz. Similarly a more up-to-date relation for the liquid water content is

$$M = 2.36 (2.705 K_{DP}/f)^{0.71} .$$

These relations are valid up to about 6 GHz¹¹. Direct extension to higher frequency (10 GHz) would produce an underestimate by about 10 %.

240 1 3 for clarity change to read: "...we consider linear orthogonal polarization in which either h or v polarized waves are transmitted, but the discussion is valid for any other orthogonal polarization basis. But the minimal depolarization of h and v polarized waves propagating in precipitation is the principal reason why the linear polarization basis is favored over circular polarization for rain rate measurements made with the WSR-88D (Doviak et al., 1998, 2000)."

240 2 To show the effects of antenna cross-polar coupling, replace this paragraph with the following:

The antenna has two ports whereby one mostly transmits and receives h polarized fields whereas the other is for v polarized fields. However, when the h or v ports are energized, v or h cross-polar radiation is also transmitted. Likewise, backscattered h or v polarized radiation can also appear in the v or h receiver channels. Cross-polar radiation from the antenna can strongly limit the accuracy of polarization measurements (Zrnić, et al. 2010)¹². To show the complications introduced by cross-polar coupling in the antenna, and to provide the basics for computing the effects of cross-polar coupling, assume the h and v ports are alternately energized every other PRT, but are simultaneously receiving both polarizations. By substituting equations (2.3) and (3.4) into (8.39), it can be shown the electric field backscattered by the n^{th} hydrometeor at r_n is

11 Ryzhkov, A., P. Zhang, J. Krause, T. Schuur, R. Palmer, and D. S. Zrnic, 2011: Simultaneous measurements of precipitation using S-band and C-band polarimetric radars. Weather Radar and Hydrology International Symposium, WRaH 2011, Exeter, UK.

12 Zrnić, D. S., R. J. Doviak, G. Zhang, and A. Ryzhkov, 2010: Bias in Differential Reflectivity due to Cross Coupling through the Radiation Patterns of Polarimetric Weather Radars. *Jo. Atmos. And Oceanic Tech.*, 27,1624-1637.

$$E_{j'}(n) = \frac{P_j^{1/2} \exp(-j2kr_n) s_{j'i_T}(n) \eta_0^{1/2} g_{i_T j}^{1/2} f_{i_T j}}{2\pi^{1/2} r_n^2}, \quad (8.42)$$

where subscripts j , i_T and $j' = v$ or h . To shorten notation we have replaced the angular weighting function $f_{i_T j}(\theta_n - \theta_0, \phi_n - \phi_0)$ with $f_{i_T j}$. The first index on g and f designates the polarization of the transmitted electric field, whereas the second index j designates the antenna's transmitting port. The first index on $s_{j'i_T}$ designates the backscattered radiation having polarization ' j' ' whereas the 2nd index designates the polarization of the transmitted wave incident on the scatterer. $E_{j'}(n)$ is the rms electric field having polarization j' backscattered by the n^{th} hydrometeor; all other terms have been defined previously. Precipitation along the path between the hydrometeor and the radar is assumed to be absent or to cause negligible attenuation and phase shifts of the propagating waves. The rms electric field, incident on the hydrometeor is Eq. (8.42) multiplied by $r_n s_{j'i_T}^{-1}(n) \exp[jkr_n]$. Thus the convention regarding.....(1975) in which $E[|s_{hh}|^2] = \sigma_b / 4\pi$. As in earlier chapters, $E[x]$ is the expected value, where 'x' is a random variable.

To show how to interpret (8.42), consider $j = j' = h$ and $i_T = h$. In this case the h port on the antenna is energized and the copolar backscattered h field is due to the antenna transmitting the copolar H field echoed by the scatterer's s_{hh} matrix element. But there is another contribution to the h backscattered field. That is, when the h antenna port is energized, a cross-polar v field is also transmitted (i.e., via the term f_{vh}), and this cross-polar field is converted by the scatterer's s_{hv} matrix element to generate an additional h copolar backscattered field. This copolar backscattered field, generated by the combination of cross-polar terms, is typically much weaker than one generated by the copolar terms because f_{vh} (as well as f_{hv} not necessarily equal to f_{vh}) and $s_{hv} = s_{vh}$ are much smaller than their corresponding copolar counterparts (i.e., f_{hh} , f_{vv} , and s_{hh} , s_{vv}). Although f_{vh} might not equal f_{hv} it can be shown reciprocity for antennas (Section 3.4) is not violated). Because this backscattered h field component is a second order product of cross-polar coupling terms (i.e., $s_{hv}f_{vh}$), it typically can be ignored and thus the h receive channel can safely be said to receive the copolar field.

But if the h antenna port is energized, first order (i.e., only a single cross-polar term is present) v backscattered fields (designated as the cross-polar field) are also generated (i.e., $j' = v$, $j = h$, and $i_T = h$ or v). This v cross-polar backscattered field is composed of two components: one generated by the product $s_{vv}f_{vh}$ and the other by $s_{vh}f_{hh}$. It is the second cross-polar component that is related to the cross-polar properties of the scatterer, but this cross-polar field can be severely corrupted by cross-polar

coupling in the antenna.

In conclusion, if the h port is energized there are four components associated with the backscattered h and v **fields**; two with the copolar h field (one is 0th order in cross polar terms, and the other is of 2nd order in cross-polar terms) and two with the cross-polar v field (both are 1st order in cross-polar terms).

3

change to read: The elemental voltage associated with the echo from the n^{th} scatterer and appearing in the h and v channels of the receiver is designated as $V_{j'i_R}(n)$, where the first index j' is the polarization of the backscattered field incident on the antenna, and the second index $i_R = \text{h or v}$ designates which *port* is receiving echoes (the second index designates the antenna port for both transmitting and receiving). Using equations (2.3), (3.20) and (3.21) it can be shown, other than for a constant receiver impedance factor,

$$V_{j'i_R}(n) = \frac{P_j^{1/2} e^{-j2kr_n} \lambda s_{j'i_T}(n) W \sqrt{g_{i_T j} g_{j'i_R} f_{i_T j} f_{j'i_R}}}{4\pi r_n^2}, \quad (8.43a)$$

where W is the range weighting function (i.e., $W(r_n - r_0)$; p. 75). If $j = i_R = \text{h}$ (i.e., the h port is transmitting and receiving) there are four components that comprise the weather signal in the copolar h receive channel. Only the $j' = \text{h}$, $i_T = \text{h}$ received signal component is 0th order in cross-coupling terms; the three others are second order and usually can be ignored. On the other hand if $j = \text{h}$, and $i_R = \text{v}$ (i.e., the h port is transmitting, but the v port is receiving cross-polar signals) there are again four components that comprise the cross-polar signals. Three of these are 1st order in cross-coupling terms one of which is the desired component related to s_{hv} . The fourth component is 3rd order in cross-coupling terms. Thus unless the antenna's cross-coupling terms f_{hv} and f_{vh} are relatively small compared to s_{hv} (i.e., $f_{\text{hv}}/f_{\text{hh}} \ll s_{\text{hv}}/s_{\text{hh}}$) measurements of s_{hv} can be significantly compromised.

If both h and v signals are transmitting simultaneously (as done with the WSR-88D), each receive channel will have 8 components of weather signals; one of 0th order (i.e., the copolar signal), three of 1st order (i.e., f_{hv} or s_{hv} only appear singularly), three of 2nd order, and one of 3rd order. Because 1st order terms are mixed with the copolar signals, f_{hv} along the antenna's copolar beam axis needs to be extremely small (i.e., less than 45 dB below the copolar peak) otherwise there can be unacceptable biases in polarimetric measurements (Zrnić, et al, 2010).

To simplify the remainder of this section, hence forth assume $g_{i_T j} = g_{j'i_R} = 0$ for $i_T \neq j$ and $j' \neq i_R$ (i.e., no cross-polar radiation is

emitted or received by the antenna) and $\sqrt{g_{ii}} f_{ii} = \sqrt{g_{jj}} f_{jj} \equiv \sqrt{g} f$ (i.e., the gain and patterns are identical for horizontal and vertically polarized waves).

241 Eq. (8.43b)

add a subscript j to $F(\mathbf{r}_n)$, and after this equation insert:
 “where $F_j(\mathbf{r}_n) = P_j^{1/2} g f^2 W \lambda / 4\pi r_n^2$. Now the first index on V_{ij} denotes the received channel (h or v) and the second index denotes the transmitted polarization (h or v). V_{ij} is a time varying voltage because scatterers reshuffle their locations due to turbulence. Thus samples of V_{ij} are obtained at each range location every PRT, although it takes two PRT intervals to collect all the polarimetric combinations of V_{ij} . The expected value of V_{ij} is zero because.....”

0 5-9

change to: “....use various second-order estimates $\langle V_{ij} V_{kl}^* \rangle_M$ of moments, to characterize the polarized signals (the subscripted angular bracket denotes an M sample average) and relate these to the properties of hydrometeors. If M is infinitely large the M sample average becomes an ensemble average equal to $E[V_{ij} V_{kl}^*]$. Because use of the angular bracket is convenient, henceforth the angular bracket without subscript, defined as an ensemble average, will be consistently shown even though radar meteorologists consistently use M sample estimates. Detailed derivation of the relationship from Eq.(8.43b) the expected value of the general term is”

(Eq. 8.44a)

at the beginning of this three part equation modify the beginning of the first line of this equation to:

$$[Co] = \langle V_{ij} V_{kl}^* \rangle = \dots\dots$$

After this equation insert: “where $[Co]$ is a 4 by 4 covariance matrix, and $F_j(\mathbf{r}_n) = F_l(\mathbf{r}_n) \equiv F(\mathbf{r}_n)$ has been assumed (i.e., the H and V weighting functions are equal). The factor $n(\mathbf{r}) \langle s_{ij} s_{kl}^* \rangle$ is called the scattering coefficient (m^{-1}).

Similar to the discussion presented on p.72 of these Supplements, the expected values are obtained by averaging both over the configuration ζ of scatterers and over the properties of the scattering matrix elements s_{ij} . Because the phases $2k(r_n - r_m)$ are uniformly distributed across 2π , the off-diagonal terms of the double sum vanish when the ensemble average is taken over ζ . Sample averages $\langle V_{ij} V_{kl}^* \rangle_M$ provide good estimates of the ensemble average if the number of independent samples M_I (Section

6.3.1.2) is sufficiently large. Because the j^{th} polarized field is assumed to be transmitted **alternately** with the l^{th} polarization, and although echoes are received simultaneously in two receivers, not all signal components are received simultaneously to obtain all the elements of $[C_o]$. For example, copolar h and copolar v signals are spaced a PRT apart. Furthermore, adjustments need to be made to the polarimetric variables (e.g., $\rho_{hv}(0)$ Section 6.8.5) to compensate for the loss of signal correlation and phase shifts due to the relative motion of scatterers during the PRT. In the last entry of Eq.(8.44a), the summation over n is replaced with.....”

241 0

At the end of this paragraph add: In this integral form $\langle s_{ij}s_{kl}^* \rangle$ are elements of the backscattering covariance matrix for a hydrometeor that represents the ensemble of hydrometeors in dV —these hydrometeors have various diameters, shapes, etc. Now the ensemble average is over the properties of the hydrometeors that cause fluctuations in s_{ij} . The expectation operation is further discussed and interpreted in the discussion of Eq. (8.45b) presented below. It might seem strange to assign the same $\langle s_{ij}s_{kl}^* \rangle$ to drops of all sizes and shapes, but it must be realized the size and shape of each scatterer is not known. The best we can do is to estimate the expected size and shape of a representative scatterer for each dV .”

1 5

change to read: “.....is a scalar multiple (through Eq. 8.43a) of the backscattering covariance matrix defined as

(8.44b)

Eq. (8.45)

to avoid confusion with the N associated with the Drop Size Distribution (DSD), and because the scatterer properties do depend on location, replace $N(\mathbf{X})$ with $p(\mathbf{X}, \mathbf{r})$. Also label this equation as (8.45a).

242 0

change $N(\mathbf{X})$ to $p(\mathbf{X}, \mathbf{r})$ and add, at the end of this paragraph, the following:

As shown in Section 8.5.2.4, the scattering matrix elements s_{ij} are functions of D_e , drop shape, canting angle, etc. Shape is related to D_e , and if the canting angle of each scatterer is zero, $p(\mathbf{X}, \mathbf{r})$ is only a function of D_e and \mathbf{r} . Under these conditions we have the simplified equation

$$p(D_e, \mathbf{r}) = \frac{N(D_e, \mathbf{r})}{\int_0^{\infty} N(D_e, \mathbf{r}) dD_e} = \frac{N(D_e, \mathbf{r})}{n(\mathbf{r})}. \quad (8.45b)$$

242 Eq. (8.46) to have consistent meaning for the parameter $\langle s_{ij}s_{kl}^* \rangle$ (see errata for page 242), the multiplicative term $n(\mathbf{r})$ needs to be inserted into the first two entries (e.g., $\langle |s_{hh}|^2 \rangle$ should be $n(\mathbf{r}) \langle |s_{hh}|^2 \rangle$).

Item 5 change to read: “Copolar correlation coefficient....”

243 0 8 to connect with discussions in Chapter 6, change to read: “...of the scatterer, and where $\delta_{hh} - \delta_{vv} = \varphi_{DS}$ (Section 6.8.2.2). If $\delta_{vv} > 0$, the phase of the scattered V field (referenced to the scatterer’s location) leads the incident V field (similar deductions can be made for all the differential phases δ_{ij}).

Eq. (8.48) change subscripts on ρ_h and ρ_v here wherever they appear to ρ_{xh} and ρ_{xv} .

2 change to read: “Analogous to....coefficients (i.e., the copolar/cross-polar correlation coefficients) can be defined as

$$\rho_{xv} = \dots\dots\dots (8.48a)$$

$$\rho_{xh} = \dots\dots\dots (8.48b)$$

The symbols ρ_{xh} and ρ_{xv} are chosen toco-polar signal (indicated by the second subscript) and cross-polar signal (indicated by ‘x’), and distinguish these.....between the copolar signals.”

244 0 3 delete “off-“ in the phrase “off-diagonal terms”

0 6 for clarification change to read as: “....phase shift φ_{DS} upon backscatter and the round trip differential phase shift $\phi_{hh} - \phi_{vv}$ due to oblate scatterers along the propagation path, it is...”

Eq.(8.49a) because we state; “...to write the correlation coefficient in which the two are separated, it would be clearer if Eqs. (8.49a,b, and c) were expressed as

$$\rho_{hv}(0) = |\rho_{hv}(0)| \exp \left[j\varphi_{DS} + j(\varphi_{hh} - \varphi_{vv}) \right] (8.49a)$$

$$\rho_{xv}(0) = |\rho_{xv}(0)| \exp \left[j\varphi_{hv}^{(b)} + j(\varphi_{vv} - \varphi_{hv}) \right] = |\rho_{xv}(0)| \exp \left[j\varphi_{hv}^{(b)} + j(\varphi_{vv} - \varphi_{hh}) / 2 \right] (8.49b)$$

$$\rho_1 = \int_0^{\infty} \frac{R(\rho)}{R(0)} d\rho$$

For the correlation function given by Eq. (10.19), ρ_0 is related to ρ_1 as

$$\rho_1 = \frac{\Gamma(v + 0.5)\Gamma(0.5)}{\Gamma(v)} \rho_0$$

- 398 Section 10.2.1: we introduce the variable $\Phi_v(\mathbf{K})$, the spatial spectrum of point velocities, in Eq.(10.46) but define it in the paragraph following its introduction (i.e., in Eq. 10.48). We should label (10.48) as (10.46), and place it before Eq.(10.46) and label it as (10.47). Other adjustments should be made to correct equation numbers; these should be few.
- 399 0 1 add the parenthetical phrase “(i.e., spectra of radial velocities along a radial and transverse to the direction of advecting turbulence)”
- 403 1 6 for a fuller explanation of the steps in Section 10.2.2, and using notation consistent with that used in earlier chapters (i.e., using $E[x]$ instead of $\langle x \rangle$ to denote the expected value), we offer the following revision of Section 10.2.2:

In this section we define the relationship between the variance of radial velocities at a *point* and the expected spectrum width measured by radar (Rogers and Tripp, 1964). Let the variance of the radial velocity $v(\mathbf{r}, t)$ at a point be σ_p^2 . This variance is the sum of the variance at all velocity scales and is defined by the equation,

$$\sigma_p^2 = E_v[v^2(\mathbf{r}, t)] - E_v^2[v(\mathbf{r}, t)] \quad (10.55)$$

where $E_v[x]$ indicates an expectation, or an average over a complete ensemble of velocity fields, all having the same statistical properties. Assume that steady wind is not present, the radar beam is fixed, and hydrometeors do not oscillate or wobble and are perfect tracers of the wind. In this case, turbulence is the only mechanism contributing to spectral broadening, and it is a random variable having zero mean (i.e., $E_v[v(\mathbf{r}, t)] = 0$).

The second central moment, σ_v^2 , of the Doppler spectrum associated with turbulence can be obtained from Eq. (5.51). Although Eq. (5.51) was derived under the assumption that $v(\mathbf{r}, t)$ is steady, this equation can be applied to the time varying wind produced by turbulence. But then σ_v^2 would be a time varying quantity because $v(\mathbf{r}, t)$ is now a time dependent variable. Replacing subscript ‘v’ with ‘t’ for pure turbulence, we obtain,

$$\sigma_v^2(t) = \sigma_t^2(t) = \overline{[v(\mathbf{r}, t) - \overline{v(\mathbf{r}, t)}]^2} = \overline{v^2(\mathbf{r}, t)} - \overline{v(\mathbf{r}, t)}^2 \quad (10.56)$$

where $\overline{\sigma_t^2(\mathbf{r}, t)}$ is the expected instantaneous second central moment of the Doppler spectrum associated with turbulence. Although $\sigma_v^2(t)$ is a function of \mathbf{r}_0 , the location of the V_6 , we have omitted \mathbf{r}_0 to simplify notation; nevertheless, the argument \mathbf{r}_0 is implicit in $\sigma_v^2(t)$. The overbar denotes a spatial average weighted by the normalized function $H_n(\mathbf{r}_0, \mathbf{r})$ where

$$H_n(\mathbf{r}_0, \mathbf{r}) = \frac{I(\mathbf{r}_0, \mathbf{r})\eta(\mathbf{r})}{\iiint I(\mathbf{r}_0, \mathbf{r})\eta(\mathbf{r})dV}$$

is a combination of reflectivity $\eta(\mathbf{r})$ and antenna pattern weights. Because the focus in this section is on the time changing velocity field we assume $\eta(\mathbf{r})$ to be time independent.

Note that $\sigma_t^2(t)$ is the expected instantaneous second central moment of the Doppler spectrum. In this case, however, the expectation, $E_\xi[x]$, is over ensembles of scatterer configurations ξ (Doviak and Zrnic, 1993, p.108) each having the same velocity field, whereas the expectation in Eq. (10.55) is taken over ensembles of velocity fields. The, expectations $E_\xi[x]$ can be made, at least in principle, over ensembles of ξ while $\hat{v}(\mathbf{r}, t_n)$ is frozen. Different scatterer configurations can be obtained by reshuffling scatterer locations. Moreover, although we can in principle freeze the velocity field, scatterers can have differential motion that results in a changing scatterer configuration. This in turn results in changes in the weather signal, and thus fluctuations of the estimates $\hat{\sigma}_t^2(t_n)$ of $\sigma_t^2(t_n)$. The circumflex $\hat{}$ denotes the estimate made from weather signal samples obtained during a dwell-time, T_d , and ' t_n ' denotes the n^{th} dwell-time, the short time span of duration MT_s , typically less than 1s. Even though estimate variance associated with different configurations of scatterers is removed by a ξ average, we retain the circumflex ' $\hat{}$ ' to emphasize the estimated value in this section pertains to one member of the ensemble of velocity fields at the n^{th} dwell-time.

The time dependence of $\hat{\sigma}_t^2(t)$ (i.e., of $E_\xi[\hat{\sigma}_t^2(t_n)]$; henceforth the expectation over ξ will not be explicitly shown) is also due to changes of turbulence on scales large compared to V_6 dimensions. Large scales of turbulence are said to create shear across V_6 . In this case large scale turbulence contributes a time varying shear component to $\hat{\sigma}_t^2(t)$ that can cause significant fluctuations of $\hat{\sigma}_t^2(t)$. Usually we are not interested in the detailed time dependence of $\hat{\sigma}_t^2(t)$, but in its statistical properties such as its mean or expected value, (i.e., $E_v[\hat{\sigma}_t^2(t_n)]$), its auto-correlation function, etc. In this section we show how $E_v[\hat{\sigma}_t^2(t_n)]$ is related to the energy density E of turbulence.

The $H_n(\mathbf{r})$ weighted radial velocity $\overline{\hat{v}(\mathbf{r}, t_n)}$ is defined as the first moment $\hat{v}_m(t_n)$ of the Doppler spectrum estimated from weather signal samples collected during T_d . The variance of $\hat{v}_m(t_n)$ is, by definition, given by

$$\text{var}[\hat{v}_m(t_n)] \equiv E_v[\hat{v}_m^2(t_n)] - E_v^2[\hat{v}_m(t_n)] \equiv \sigma_v^2(t_n). \quad (10.57a)$$

For pure turbulence, $E_v[\hat{v}_m(t_n)] = 0$ and thus

$$\sigma_v^2(t_n) = E_v[\hat{v}_m^2(t_n)]. \quad (10.57b)$$

Because (10.56) applies to one member of an ensemble of velocity fields, we express (10.56) as

$$\hat{\sigma}_t^2(t) = \overline{[\hat{v}(\mathbf{r}, t) - \bar{\hat{v}}(\mathbf{r}, t)]^2} = \overline{\hat{v}^2(\mathbf{r}, t) - \bar{\hat{v}}(\mathbf{r}, t)^2}$$

Where the diacritical simply emphasizes that $\hat{\sigma}_t^2(t)$ is an estimate for one T_d which essentially samples the velocity field. By taking the velocity ensemble average of this equation and substituting Eq. (10.57b) into it, we obtain, after commuting ensemble and spatial averages (i.e.,

$$E_v[\overline{v^2(\mathbf{r}, t_n)}] \equiv \overline{E_v[v^2(\mathbf{r}, t_n)]},$$

$$E_v[\overline{\hat{\sigma}_t^2(t_n)}] + \sigma_v^2(t_n) = \overline{E_v[\hat{v}^2(\mathbf{r}, t_n)]}. \quad (10.58a)$$

The weighted spatial average of $E_v[\hat{v}^2(\mathbf{r}, t_n)]$ is, by definition,

$$\overline{E_v[\hat{v}^2(\mathbf{r}, t_n)]} = \int_V E_v[\hat{v}^2(\mathbf{r}, t_n)] H_n(\mathbf{r}_0, \mathbf{r}) dV \quad (10.58b)$$

The derivation leading to Eq. (10.58) does not require turbulence to be statistically stationary, homogeneous, or isotropic. That is, Eq. (10.58a) relates the expected value of the second central moment of measured Doppler spectra (i.e., measured estimated with short dwell-times), and the variance of the mean Doppler velocity, to the $H_n(\mathbf{r}_0, \mathbf{r})$ weighted spatial average of the expected value of the second central moment of the radial component of turbulence at each point \mathbf{r} and t_n . This is in agreement with results of Rogers and Tripp (1964).

These results apply to estimates of the Doppler velocity and second central moments made with any dwell-time. If longer dwell-times are used, $E_v[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)}]$ increases because velocity components associated with large scale turbulence have time to evolve within the resolution volume and be adequately sampled. On the other hand, the variance σ_v^2 of the mean Doppler velocity decreases as dwell time increases, and it vanishes in the limit of an infinite dwell time. In this limit, $\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)}$ solely measures the spatial average of the weighted distribution of turbulence at each point.

10.2.2.1 Estimate variance due to changes in scatterer configuration

It should be noted that the variance, $\sigma_v^2(t_n)$, does not include the variance associated with the statistical uncertainty due to changing scatterer configurations. Nevertheless, variance (e.g., that discussed in Chapter 6) associated with the weather signal fluctuations due to changes in the scatterer configuration can be significant, and it needs to be included in any rigorous analysis of radar measurements of turbulence.

For example, in addition to the variance of $\overline{\hat{v}(\mathbf{r}, t_n)}$ due to the time changing velocity field, we have additional variance associated with the random location of scatterers (i.e., the time dependence of the true $\overline{\hat{v}(\mathbf{r}, t_n)}$ differs from the time dependent $\overline{\hat{v}(\mathbf{r}, t_n)_R}$ estimated with radar). Here $\overline{\hat{v}(\mathbf{r}, t_n)}$ is the weighted velocity irrespective of the scatterer configuration.

Even if $\hat{v}(\mathbf{r}, t_n)$ was a constant independent of time, the radar estimates $\overline{\hat{v}(\mathbf{r}, t_n)_R}$ would be fluctuating due to the fact that scatterers move continuously to new locations for the same velocity field. That is, there is an evolving configuration of scatterers, and each configuration of scatterers produces a different weather signal sample from which $\overline{\hat{v}(\mathbf{r}, t_n)}$ is estimated. In general, time fluctuations of estimates are due to both a changing velocity field and a changing configuration of scatterers.

To illustrate, assume a constant wind that carries scatterers along range arcs. In this case, the radial velocity field $\overline{\hat{v}(\mathbf{r}, t_n)} = 0$. Nevertheless, radar estimates of $\overline{\hat{v}(\mathbf{r}, t_n)}$ are time varying and random; this is so because the scatterers' configuration within V_6 continually changes as new scatterers enter V_6 , and others leave it. That is, the In-phase, I , and Quadrature, Q , components of the weather signal are still Gaussian distributed random variables as shown in Fig. 4.4a. In other words, although the mean or expected Doppler velocity is zero, the time sequence of the I , Q , samples will randomly move in the I , Q plane, and Doppler velocity estimates made with a small number of samples (e.g., two) can have non zero values.

The changes of I , Q from sample pair to sample pair can be relatively small if the sample pair spacing is short compared to the correlation time τ_c of the weather signals, and if the intra-pulse spacing $T_s \ll \tau_c$. The weather signal correlation time τ_c , equal to the time required to flush V_6 with new scatterers, is not necessarily equal to the correlation time of the velocity field; in our simple illustration the correlation time of the velocity field is infinite. The non zero velocity estimates, calculated from pairs of I , Q samples, are uncorrelated if the pair spacing is longer than τ_c . Only with a long time average will these velocity estimates average to 0.

These arguments, applied to the radar estimates of $\overline{\hat{v}(\mathbf{r}, t_n)}$, can also be applied to show that the ξ expectation of the radar estimates $\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)_R}$ {i.e., $E_\xi \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)_R} \right]}$ equals $\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)}$.

10.2.2.2 Homogeneous turbulence

If turbulence is homogeneous over the region where the weighting functions contribute significantly (i.e., turbulence is locally homogeneous), Eq. (10.58b) shows that $E_v[\overline{\hat{v}^2(\mathbf{r}, t)}] = \sigma_p^2(t_n)$, the ‘‘point-measured variance’’ (Frisch and Clifford, 1974; the following paragraph will clarify what is meant by ‘‘point-measured variance’’). The radial component of the turbulent energy density at a point is, $E_r = \frac{1}{2} \gamma \sigma_p^2$, where γ is the air mass density. Using Eq. (10.58a), and noting that $E_v[\overline{\hat{v}^2(\mathbf{r}, t_n)}] = \sigma_p^2(t_n)$, we can then relate E_r to radar measurements as

$$E_r = \frac{1}{2} \gamma \sigma_p^2(t_n) = \frac{\gamma}{2} \left\{ E_v \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)} \right] + \sigma_v^2(t_n) \right\}. \quad (10.59)$$

If turbulence is isotropic, the total turbulence energy density $E = 3 E_r$.

Eq. (10.59), establishes a relation between the radial component of the “point-measured” turbulent energy density and the second central moment of the Doppler spectrum associated with turbulence, but it requires turbulence to be *locally* homogeneous although not isotropic or stationary. Therefore, the “point” under discussion is, in reality, a collection of points over the entire resolution volume wherein turbulence is assumed to have the same statistical properties at each point. Section 10.2.2.3 presents results for the case where turbulence is inhomogeneous.

Eq. (10.59) demonstrates that the energy density of the radial component of “point-measured” homogeneous turbulence can be calculated from the sum of the expected value of the second central moment, $E_v \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)} \right]$, and the variance, $\sigma_v^2 = E_v \left[\overline{\hat{v}(\mathbf{r}, t_n)^2} \right]$, of the mean Doppler velocity estimates. It also shows how that energy is partitioned between large and small scales of turbulence; σ_v^2 is principally due to large scale turbulence whereas $E_v \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)} \right]$ is principally due to small scale turbulence.

If the radial component of turbulence has a -5/3rds power law vs wavenumber $K = 2\pi / \Lambda$ for all wavelengths Λ of the spectrum of turbulence, and if the dimensions of V_6 are the same in all directions (i.e., $\sigma_\theta r_0 = \sigma_\phi (\theta_0) r_0 = \sigma_r$; Section 5.3), it can be shown that turbulence from all $\Lambda \leq L_c \equiv \sigma_\theta r_0$, the characteristic size of V_6 , contributes only about 20% to $E_v \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)} \right]$. Thus, although some portion of $E_v \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)} \right]$ is due to small scales, most of its contribution comes from turbulence on wavelengths large compared to L_c ; this appears at variance with previously published interpretations. For example, if the weighting function is uniform, as stipulated by Rogers and Tripp (1964), across V_6 having dimensions $LxLxL$, only 36% of $E_v \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)} \right]$ is due to turbulence from all $\Lambda \leq L$. This contradicts Rogers and Tripp (1964) statement that $E_v \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)} \right]$ “receives spectral contributions mainly from the wavelengths shorter than the dimensions of V_6 ”.

Large scale (i.e., large compared to V_6 dimensions) turbulence shifts the Doppler spectrum along the velocity axis so that the single spectrum mean Doppler velocity changes from one spectrum to the next. Thus to estimate $E_v \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)} \right]$ we need to average the second central moments calculated about each of the fluctuating means. For stationary and/or globally homogeneous turbulence, expectations can be obtained from averages over time and/or space (i.e., at different \mathbf{r}_0 locations).

10.2.2.3 Inhomogeneous turbulence

It is not necessary to assume turbulence is homogeneous (as we did in arriving at Eq. 10.59) to obtain a relation between the point variance of the radial component of turbulence and radar measurements. If turbulence is not homogeneous, $\sigma_p^2(\mathbf{r}, t_n)$ is still the variance at a point \mathbf{r} , but $\overline{\sigma_p^2(\mathbf{r}, t_n)}$ is the $H_n(\mathbf{r}_0, \mathbf{r})$ weighted spatial average wind variance at each point. Then the expression for the point variance must be written as,

$$\overline{\sigma_p^2(\mathbf{r}, t_n)} = E_v \left[\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)} \right] + \sigma_v^2(t_n).$$

This is exactly the same form as Eq. (10.59), but we now have an overbar on $\sigma_p^2(\mathbf{r}, t_n)$. This simply means that radar can only measure the $H_n(\mathbf{r}_0, \mathbf{r})$ weighted spatial average of turbulence at each and every point.

As stated earlier (Section 10.2.2.1), the variance σ_v^2 does not include the variance associated with the statistical uncertainty of the estimates of $\overline{v(\mathbf{r}, t)}$ due to weather signal fluctuations (i.e., the variance associated with changes in the scatterers' configuration). The variance associated with the statistical uncertainty of the estimates must be subtracted from the measured variance in order to obtain σ_v^2 ; window biases, typically associated with the measurements of $\overline{\hat{\sigma}_t^2(\mathbf{r}, t_n)}$ (Doviak and Zrnic, 1984, Fig. 6.8; Melnikov and Doviak, 2002), must also be taken into account.

- 430 Change the title of Section 11.3 to: “Solution for Fields Scattered by Refractive Index Perturbations” and everywhere else “irregularities” appear, change this word to “perturbations”
- 1 5-6 Change “spectrum of turbulence” to “spatial spectrum of refractive index perturbations”
- 2 2 after “links,” insert “and because enhanced Δn is typically confined to thin layers,”
- Fig.11.2 change figure caption to read: “...of the scattering geometry in a vertical plane containing T, R, and the spheroidal.....spheroid, \mathbf{a}_{t0} and \mathbf{a}_{r0} are unit vectors at the origin defined by the intersection of the transmitting and receiving beam axes. Vectors are indicated.....”
- Fig.11.2 A comment on bistatic radar measurements:

Fig.11.2 would be more applicable to dual-Doppler radar remote sensing (i.e., measuring the Doppler shift of echoes received at T and at R) to map winds if the width of a narrow transmitting beam (e.g., $\approx 1^\circ$) was shown. For wind mapping the receiver beam is often much broader and fixed to receive bistatic echoes from anywhere the transmitting beam intersects scatterers. The location of the scatterers along the transmitting beam is determined by the arrival time of signals received at R. Thus, if the refractive perturbations are spatially uniform, the bistatic scattering volume ($V_6^{(b)}$; shaded area) would be bounded by the volume common to the spheroidal shell of thickness Δs and the transmitter beam. In this case the elemental scattering volume dV would be confined to V_B where the origin of $V_6^{(b)}$ is at the intersection of the transmitter's beam axis with the spheroidal surface centered within the shell of thickness Δs defined by Eq. (11.23). Thus there is a continuous distribution of $V_6^{(b)}$ along the transmitted beam, and the location of each $V_6^{(b)}$ is determined by the time signals are received at R.

431 2 15, 16 change to read: "...The range r_{r0} from the receiver to $V_6^{(b)}$ is" and in Eq. (11.22) change r_r to r_{r0} .

18 change to "where θ_r is not the receiving beam's elevation angle, but the elevation angle to $V_6^{(b)}$, and $2d$ is the"

20 modify to: "...pulse is radiated, and it determines the location of the spheroidal surface centered within the shell of thickness Δs . Thus t_s determines the locations of spheroidal shells along the transmitter beam. Obviously....."

443 Section 11.4.3 to differentiate the commonly known Bragg scatter associated with steady or deterministic perturbations from that Bragg scatter associated with random perturbations, we introduce the term "Stochastic Bragg Scatter" by replacing the second sentence of this section with:

“Perturbations in atmospheric refractive index are caused by temperature and humidity fluctuations; thus the perturbation in n is a random variable having a spectrum of scales. Although there is a spectrum of spatial scales, only those at about the Bragg wavelength $\Lambda_B = \lambda/[2\sin(\theta_s/2)]$ contribute significantly to the backscattered power. Because scatter is from spatial fluctuations in refractive index, the scattering mechanism is herein defined as Stochastic Bragg Scatter (SBS). Because there are temporal fluctuations as well, the scattered power is also a random variable and its properties are related to the statistical properties of the scattering medium. In this section we relate the expected....(return to the 3rd sentence in the text)”

459 Eq. (11.124) this equation assumes that the beam width is given by Eq. (3.2b). A more general form is

$$\rho_{\perp} = \frac{D_a \sqrt{2}}{\pi \gamma_1}, \quad \theta_1 = \gamma_1 \frac{\lambda}{D_a}$$

4 at the end of this paragraph, “...in this section.”, add: “Under far field conditions the beamwidth part of the “resolution volume weighting” term in Eq.(11.122) does not contribute significantly to the integral, but beamwidth and range resolution do contribute to the backscattered power because they multiply the integral in Eq.(11.122).”

460 0 2 add the following sentence at the end of the line:
 ρ_h is the outer scale of the refractive index irregularities, but condition (11.124) applies to the transverse correlation lengths of the Bragg scatterers. Thus, the conclusion reached in this paragraph applies if the Bragg scatterer’s correlation length equals the outer scale.

1 49 the following comments offer an alternative and hopefully better explanation: The angular width (i.e., $\approx \lambda/\rho_B$) of the diffraction pattern for each Bragg scatterer narrows as the correlation length of the Bragg scatterer increases, but reaches a limit $\lambda/2r_F$, where r_F is the first Fresnel zone radius. Because the diffraction pattern’s main lobe is centered about the forward scatter direction (i.e., horizontally extended Bragg scatterers displaced horizontally from above the radar, have most significant radiation in a forward scatter direction such that the angle of reflection is equal to the angle of incidence), only those Bragg scatterers within $\lambda z_o/\rho_B$ can contribute significantly to the receiver. The Fresnel term accounts for this diminished contribution from Bragg scatterers.

461 0 11 insert after “...in space.”: “This is a consequence of the greater importance of the Fresnel term relative to the resolution volume weighting term (i.e., in Eq.11.122) along the transverse directions.”

478 0 7 rewrite the sentence beginning with “Then g is related...” as: “If the radiation pattern is well defined by a Gaussian function, g (now the directivity, Section 3.1.2) is related to θ_1 as...”

Appendix A.2 for clarification and consistency with notations used in Section 2.2.3, make the following revisions:

1 2 “...relate scatterer height h_s to radar”. Also change h to h_s everywhere in Appendix A.2 if it refers to the height of the scatterer (e.g., the upper limit of integration in Eqs. (A.8) and (A.9).

2 1-10 In a time interval τ_s (range-time; Section 3.4.2) a transmitted microwave pulse travels along a path to a scatterer at range l (in chapter 2 we use ‘ r ’ for true range) and returns as a microwave pulse (echo) of considerably reduced power. Range-time is the time delay, after the time a microwave pulse is transmitted, that an echo is received (Section 3.4.2). If the refractive index n is uniform along the path, the path is straight and the two-way distance is $2l = c\tau_s / n$. But if n is a function of distance l along the path, the path is curved and the differential length dl at any point along the path is

$$dl = cd\tau_s / 2n(l)$$

Radar meteorologists typically assign an apparent range r_a to the scatterer assuming the pulse travels at the constant speed c of light in free space. Thus the apparent differential range is

$$dr_a = cd\tau_s / 2 = n(l)dl, \text{ and thus } r_a = \int_0^l n(l)dl$$

is the relation between the true path length l (i.e., true range r) of the scatterer and the radar measured apparent range r_a .

3 1 for consistency and clarity, modify to “.....deduce that $dh = dR = dl \sin \theta$, so that...

8 modify to read: “...substitute h_s for the upper integration limit in Eq. (A.5a) to obtain...”

513 3 4 rewrite as: “...independent of all others because the shell is assumed to be many wavelengths thick and scatterers are randomly placed in the shell.

547 Index add: “Antenna; far field, 435-436, 459”

548 Index add: “Bright band, pp. 256, 268”

554 Index add “Melting layer, pp. 225, 255”

556 Index for the entry “Radome losses” add page 43.