

## Repetitive Decision Making and the Value of Forecasts in the Cost-Loss Ratio Situation: A Dynamic Model

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### ABSTRACT

The purposes of this paper are to describe a dynamic model for repetitive decision-making in the cost-loss ratio situation and to present some theoretical and numerical results related to the optimal use and economic value of weather forecasts within the framework of the model. This model involves the same actions and events as the standard (i.e., static) cost-loss ratio situation, but the former (unlike the latter) is dynamic in the sense that it possesses characteristics (e.g., decisions, events) that are related over time. We assume that the decision maker wants to choose the sequence of actions over an  $n$ -occasion time period that minimizes the total expected expense. A computational technique known as stochastic dynamic programming is employed to determine this optimal policy and the total expected expense.

Three types of weather information are considered in studying the value of forecasts in this context: 1) climatological information; 2) perfect information; and 3) imperfect forecasts. Climatological and perfect information represent lower and upper bounds, respectively, on the quality of all imperfect forecasts, with the latter considered here to be categorical forecasts properly calibrated according to their past performance. Theoretical results are presented regarding the form of the optimal policy and the relationship among the total expected expenses for these three types of information. In addition, quality/value relationships for imperfect forecasts are described.

Numerical results are derived from the dynamic model for specific values of the model parameters. These results include the optimal policy and the economic value of perfect and imperfect forecasts for various time horizons, climatological probabilities, and values of the cost-loss ratio. The relationship between the accuracy and value of imperfect forecasts also is examined.

Several possible extensions of this dynamic model are briefly discussed, including decision-making problems involving more actions and/or events, more complex structures of the costs and losses, and more general forms of imperfect forecasts (e.g., probability forecasts).

### 1. Introduction

Many decision-making problems that are sensitive to weather information are repetitive in nature, in the sense that they involve identical or similar decisions made on a sequence of occasions. The frost-protection and fallowing-planting problems are two examples of such situations (e.g., Katz *et al.*, 1982; Brown *et al.*, 1984). These situations generally possess characteristics that are related over time; that is, the current decision depends on previous decisions and

past weather events and, in turn, affects future decisions and their consequences. Clearly, repeated application of a static decision-making model would not be appropriate in such situations. Instead, a dynamic model is required in order to take relationships among the relevant characteristics into account.

Since 1950, meteorologists have devoted considerable attention to a simple decision-making problem commonly referred to as the "cost-loss ratio situation" (e.g., Thompson, 1952, 1962; Thompson and Brier, 1955; Murphy, 1977). In essence, this problem involves a decision maker who must decide whether or not to protect an activity in the face of uncertainty as to whether or not adverse weather will occur. In

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formulating decision-making models in the cost-loss ratio situation, it has been assumed heretofore that either 1) the decision is made on a single isolated occasion or 2) similar decisions are made on two or more occasions but the characteristics of the situations on these occasions are unrelated. Since repetitive decision-making situations generally do not satisfy the latter assumption (as noted earlier), it would be desirable to have a dynamic model suitable for application in this context.

The primary purposes of this paper are 1) to describe a dynamic model for repetitive decision making in the cost-loss ratio situation and 2) to present some theoretical and numerical results related to the optimal use and economic value of weather forecasts within the framework of this model. Section 2 contains a description of the dynamic model and the types of forecasts of interest here. This section also illustrates the model using a two-occasion situation. Section 3 describes the general solution procedure for this dynamic decision-making problem and summarizes theoretical relationships between the economic value of different types of forecasts. Numerical results derived from the dynamic model are presented in Section 4, including optimal strategies for the decision maker and the economic value of forecasts for various values of the model parameters. Section 5 consists of a discussion and conclusion. The derivations of some theoretical results summarized in Section 3 are included in two appendices.

## 2. The dynamic model

### a. Basic model and assumptions

We are concerned here with an individual who must decide on each of  $n$  occasions (e.g., days) whether or not to protect an activity in the face of uncertainty as to whether or not weather adverse to the activity will occur on that occasion or on future occasions. Specifically, this situation involves two actions (protect, do not protect) and two events (adverse weather, no adverse weather). The period over which these decisions must be made is referred to as the "horizon." A cost  $C$  is incurred on each occasion that protective action is taken, and it is assumed that taking protective action precludes incurring any losses on that occasion (i.e., the activity is completely protected). If protective action is not taken and adverse weather occurs, then the decision maker suffers a loss  $L$ . It is assumed that this loss can be incurred at most once (i.e., it is a complete loss from which no recovery is possible) and that  $0 < C < L < \infty$ . Finally, if protective action is not taken and adverse weather does not occur, then no loss is incurred.

The individual or decision maker is assumed to want to minimize the *total expected expense* over the entire  $n$ -occasion horizon, and thus he/she will choose

the strategy or sequence of actions (i.e., protect, do not protect) over this period that will lead to such a minimum.<sup>1</sup> Total expected expense is the probability-weighted average of the expenses that might be incurred during the  $n$ -occasion horizon. The criterion of minimizing total expected expense—or, equivalently, maximizing total expected return—requires that the decision maker be risk neutral (i.e., that the decision maker's utility function be linear in monetary expense; see Winkler and Murphy, 1985).

### b. Types of, expense of, and value of forecasts: Some definitions

Three types of information or forecasts are of interest here: 1) climatological information; 2) imperfect forecasts; and 3) perfect information. In order to describe these types of information in appropriate detail, it is convenient to introduce the following notation:

#### (i) Events and climatological probabilities—

$\theta$  is a random variable denoting the occurrence ( $\theta = 1$ ) or nonoccurrence ( $\theta = 0$ ) of adverse weather;  
 $p_\theta = P\{\theta = 1\}$  is the climatological (e.g., long-term historical or "prior") probability of adverse weather.

#### (ii) Forecasts—

$Z$  is a random variable denoting a (categorical) forecast of adverse weather ( $Z = 1$ ) or no adverse weather ( $Z = 0$ );

$p_Z = P\{Z = 1\}$  is the probability of a forecast of adverse weather;

$p_1 = P\{\theta = 1 | Z = 1\}$  is the conditional (or "posterior") probability of adverse weather given a forecast of adverse weather;

$p_0 = P\{\theta = 1 | Z = 0\}$  is the conditional (or "posterior") probability of adverse weather given a forecast of no adverse weather.

The quality of the imperfect forecasts is fully described by the two conditional probabilities  $p_1$  and  $p_0$ . Without any loss of generality, it can be assumed that  $p_0 < p_\theta < p_1$  (this relationship implies that adverse weather is more likely to be preceded by a forecast of adverse weather than by a forecast of no adverse weather). Then, from the definition of conditional probability,

$$p_Z = \frac{p_\theta - p_0}{p_1 - p_0}. \quad (1)$$

Hence, only three parameters— $p_\theta$ ,  $p_0$ , and  $p_1$ —are needed to characterize the imperfect forecasts. Since the probabilities  $p_1$  and  $p_0$  indicate the likelihood of

<sup>1</sup> It will be demonstrated by example in Section 2c that the naive strategy of minimizing the immediate expected expense on each individual occasion is not equivalent to minimizing the total expected expense over the entire time period.

occurrence of adverse weather given categorical (i.e., nonprobabilistic) forecasts of adverse weather and no adverse weather, respectively, the imperfect forecasts can be considered to be categorical forecasts properly calibrated according to their past performance (i.e., adverse weather has occurred a proportion  $p_1$  of the occasions in the past with a forecast of adverse weather and a proportion  $p_0$  of the occasions in the past with a forecast of no adverse weather). Alternatively, this formulation of imperfect forecasts can be viewed as a special case of probability forecasts in which only two probabilities,  $p_0$  and  $p_1$ , are used (cf. Winkler *et al.*, 1983).

Climatological information corresponds to the limiting case of imperfect forecasts for which  $p_1 = p_0 = p_\theta$ . We shall denote the minimum total expected expense over an  $n$ -occasion horizon associated with climatological information and imperfect forecasts by  $E_C(n)$  and  $E_F(n)$ , respectively. Moreover,  $E_C(n)$  is used in this paper as a benchmark against which to compare the minimum total expected expense associated with other types of information. Thus, if  $V_F(n)$  denotes the value of imperfect forecasts over the  $n$ -occasion horizon, then

$$V_F(n) = E_C(n) - E_F(n). \tag{2}$$

Perfect information, on the other hand, corresponds to the limiting case of imperfect forecasts for which  $p_1 = 1$ ,  $p_0 = 0$ , and necessarily  $p_Z = p_\theta$ . If  $E_P(n)$  denotes the minimum total expected expense over the  $n$ -occasion horizon associated with this type of information, then

$$V_P(n) = E_C(n) - E_P(n). \tag{3}$$

Perfect forecasts are of interest here because  $E_P(n)$  and  $V_P(n)$  represent lower and upper bounds, respectively, on the expected expense and value of all imperfect forecasts. Relationships among these quantities are discussed in greater detail in Section 3d.

*c. An illustration: The two-occasion (n = 2) horizon*

For simplicity, we consider here a two-occasion ( $n = 2$ ) horizon. The sequential decision-making problem in this situation is depicted in the form of a decision tree in Fig. 1. In this tree, decision points (or nodes) are denoted by squares and event nodes are denoted by circles. At each decision node the decision maker must decide whether to protect ( $A = 1$ ) or not to protect ( $A = 0$ ), and at each event node the uncertainty concerning the events is resolved by the occurrence of adverse weather ( $\theta = 1$ ) or no adverse weather ( $\theta = 0$ ). The probability of adverse weather is taken here to be the climatological probability  $p_\theta$ ; that is, in this example we are considering only the case of climatological information.

A sequential decision-making problem is analyzed by starting with the final-period decision and working

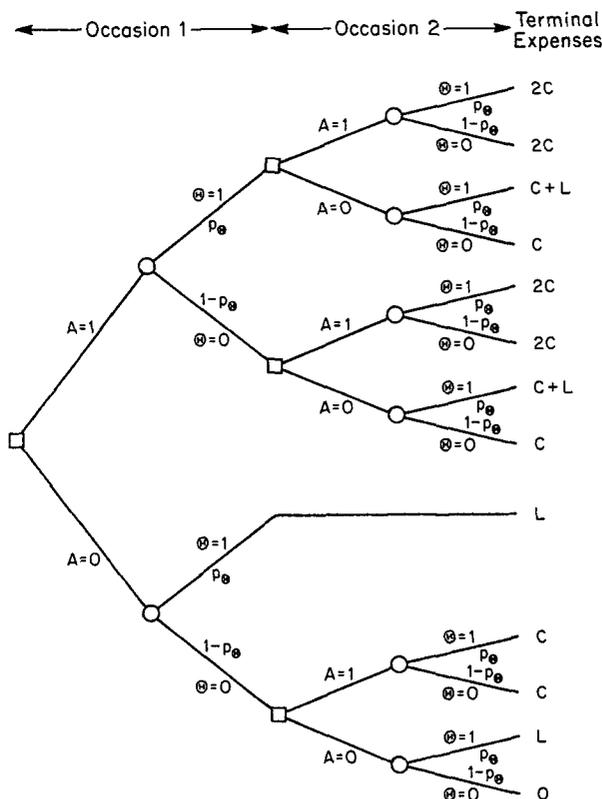


FIG. 1. Decision tree for two-occasion ( $n = 2$ ) dynamic cost-loss ratio situation, with climatological information.

back to the initial-period decision. This process is known as “backward induction,” or “averaging out and folding back” (e.g., Raiffa, 1968). In the situation represented in Fig. 1, the decision on the second occasion is analyzed first. But since occasion 2 is the final occasion, this decision is a single-period decision which is uninfluenced by any future decisions. Therefore, the optimal action on occasion 2 is dictated by the result obtained for the static cost-loss ratio model; namely, protect ( $A = 1$ ) if  $p_\theta > C/L$  and do not protect ( $A = 0$ ) if  $p_\theta < C/L$ . Knowing that this strategy will be optimal on occasion 2, we can move back to the initial decision on occasion 1.

First, suppose that  $p_\theta > C/L$ , which means that the preferred decision will always be to protect on occasion 2 (unless, of course, there is no choice on occasion 2 because the loss  $L$  was incurred on occasion 1). As a result, if the decision maker protects on occasion 1, the overall expected expense will be  $2C$  (the decision maker will protect on both occasions and a cost  $C$  will be incurred on each occasion). If the decision maker does not protect on occasion 1, the overall expected expense will be  $p_\theta L + (1 - p_\theta)C$  [with  $p_\theta L$  corresponding to the probability  $p_\theta$  of suffering the loss  $L$  on occasion 1 and  $(1 - p_\theta)C$  corresponding to the probability  $1 - p_\theta$  of no adverse weather occurring on the first occasion and then protecting on occasion

2]. Thus, the decision maker should protect on occasion 1 if

$$2C < p_{\theta}L + (1 - p_{\theta})C,$$

which simplifies to

$$p_{\theta} > C/(L - C).$$

Note how the anticipation of the second-occasion decision changes the critical value of  $p_{\theta}$  on the first occasion from  $C/L$  to a larger value  $C/(L - C)$ . If  $C/L < p_{\theta} < C/(L - C)$ , then the decision maker should not protect on occasion 1 but should plan to protect on occasion 2 (if adverse weather does not occur on occasion 1, of course). If  $p_{\theta} > C/(L - C)$ , then the probability of adverse weather is high enough to warrant taking protective action on both occasions. It may be of interest here to note that minimizing *immediate* expected expense—a strategy consistent with repeated application of the static model—yields a total expected expense of  $2C$  in this case. However, this expected expense is not necessarily the minimum expected expense over the two occasions [if  $C/L < p_{\theta} < C/(L - C)$ , then the minimum expected expense is  $p_{\theta}L + (1 - p_{\theta})C$ , which is less than  $2C$ ].

Next, consider the case in which  $p_{\theta} < C/L$ . On occasion 2, the preferred decision should be not to protect. Moving back to occasion 1, the decision maker finds that protecting on occasion 1 leads to an overall expected expense of  $C + p_{\theta}L$  ( $C$  for protecting on the first occasion and  $p_{\theta}L$  for the probability of suffering the loss  $L$  on the second occasion, when no protection will be used). Not protecting on occasion 1 yields an overall expected expense of  $p_{\theta}L + (1 - p_{\theta})p_{\theta}L$  [ $p_{\theta}L$  corresponding to the probability  $p_{\theta}$  of suffering the loss on occasion 1 and  $(1 - p_{\theta})p_{\theta}L$  corresponding to the probability  $p_{\theta}(1 - p_{\theta})$  of surviving the first occasion but suffering the loss on the second occasion]. On occasion 1, then, the decision maker should protect if

$$C + p_{\theta}L < p_{\theta}L + (1 - p_{\theta})p_{\theta}L,$$

which reduces to

$$p_{\theta}(1 - p_{\theta}) > C/L.$$

But this inequality cannot hold since we have assumed in this case that  $p_{\theta} < C/L$ , which implies that

$$p_{\theta}(1 - p_{\theta}) < p_{\theta} < C/L.$$

Therefore, the decision maker should not protect on either occasion if  $p_{\theta} < C/L$ .

An alternative way to analyze sequential problems is to evaluate overall strategies, where a strategy consists of a sequence of choices on the occasions of concern (this approach is not feasible for the general  $n$ -occasion problem). For example, in the two-occasion sequential cost-loss ratio situation, there are four possible overall strategies: protect on both occasions,

protect on occasion 1 but do not protect on occasion 2, do not protect on occasion 1 but protect on occasion 2 if  $L$  is not incurred on occasion 1, and do not protect on both occasions. These four strategies are listed in Table 1 along with their total expected expenses and the values of  $p_{\theta}$  for which each strategy is optimal. Each total expected expense is determined by taking the terminal expenses associated with the branches corresponding to the strategy and weighting these expenses by the probabilities of reaching these particular end points. The values of  $p_{\theta}$  for which each strategy is optimal agree with the results obtained in the sequential analysis; for example, under no circumstances should the decision maker protect on occasion 1 but not on occasion 2.

For a numerical example, suppose that  $C = \$30$  and  $L = \$100$ . Then the decision maker should protect on occasion 1 if  $p_{\theta} > C/(L - C) = 0.429$ , as compared with protecting if  $p_{\theta} > C/L = 0.300$  under the static model. Moreover, if we assume that  $p_{\theta} = 0.400$ , then the decision maker should not protect on occasion 1 but should protect on occasion 2. The total expected expense associated with this strategy is  $p_{\theta}L + (1 - p_{\theta})C = \$58$ , which is less than the total expected expense associated with the strategy of protecting on both occasions (namely,  $2C = \$60$ ). The general solution to the  $n$ -occasion sequential cost-loss ratio problem is described in Section 3.

### 3. Solution procedures and theoretical results

Expressions for the total expected expenses of the various types of weather information are needed to obtain quantitative estimates of the value of weather forecasts in the dynamic cost-loss ratio decision-making situation. In this section a computational method is described for determining the optimal actions and associated minimum total expected expenses for the general  $n$ -occasion time period decision-making problem. Some theoretical results also are presented concerning the nature of the optimal actions and the relationship between the quality and value

TABLE 1. Total expected expenses for climatological information and values of  $p_{\theta}$  for which respective strategies are optimal in two-occasion sequential cost-loss ratio situation.

Strategy	Action		Total expected expense	Values of $p_{\theta}$ for which strategy is optimal
	Occasion 1	Occasion 2		
1	$A = 1$	$A = 1$	$2C$	$C/(L - C) < p_{\theta} < 1^*$
2	$A = 1$	$A = 0$	$C + p_{\theta}L$	None**
3	$A = 0$	$A = 1$	$p_{\theta}L + (1 - p_{\theta})C$	$C/L < p_{\theta} < C/(L - C)$
4	$A = 0$	$A = 0$	$[1 - (1 - p_{\theta})^2]L$	$0 < p_{\theta} < C/L$

\* Since  $C/(L - C) > 1$  if  $C/L > 1/2$ , strategy 1 is never optimal if  $C/L > 1/2$ .

\*\* Since  $p_{\theta}L + (1 - p_{\theta})C \leq C + p_{\theta}L$  for all values of  $p_{\theta}$ , strategy 2 is never optimal.

of weather forecasts. The derivations of these theoretical results are included in Appendices A and B.

The dynamic cost-loss ratio situation constitutes a special case of a general class of dynamic decision-making models known as *Markov decision processes* (Ross, 1970, Chap. 6). For such a class of models, a computational technique known as *stochastic dynamic programming* (White, 1979; Ross, 1983) can be employed to determine the optimal actions and total expected expenses. Dynamic programming is based on the concept of "backwards induction" introduced in the illustration of the special case of a two-occasion (i.e.,  $n = 2$ ) time period in Section 2c. This technique has been employed to quantify the value of weather forecasts for a more complex dynamic decision-making model representing the so-called fruit-frost situation (Katz *et al.*, 1982; Stewart *et al.*, 1984).

We note that it is possible to derive analytical expressions for the minimum total expected expenses for each of the different types of weather information for the general  $n$ -occasion time period problem. However, such expressions are quite complicated and the dynamic programming technique has the additional advantage that it can be readily adapted to handle generalizations of the dynamic cost-loss ratio model. Such generalizations might include changing the nature of the dynamics (e.g., by assuming proportionate losses rather than a single complete loss) or changing the nature of the weather forecasts (e.g., by considering forecasts that are presented in probabilistic terms).

*a. Climatological information*

We let  $E_C(k)$  denote the minimum total expected expense for the last  $k$  occasions of a  $n$ -occasion time period decision-making problem ( $k = 1, 2, \dots, n$ ) when only climatological information is available and given that no loss  $L$  has yet been incurred. For the  $k = 1$  case (i.e., the static cost-loss ratio situation), it is easy to show (as observed in Section 2c) that

$$E_C(1) = \min(p_\theta L, C). \tag{4}$$

Then for the  $k = 2$  case, using the fact that the  $k = 1$  case already has been solved with  $E_C(1)$  given by (4),

$$E_C(2) = \min[(1 - p_\theta)E_C(1) + p_\theta L, C + E_C(1)]. \tag{5}$$

The first term on the right-hand side (rhs) of (5) represents the total expected expense over the last two occasions when protective action is not taken on the next to last occasion. With probability  $1 - p_\theta$ , adverse weather will not occur on the next to last occasion and the total expected expense will be the expected expense on the last occasion; with probability  $p_\theta$ , the loss  $L$  will be suffered on the next to last occasion. The second term on the rhs of (5) represents the total expected expense when protective action is taken on the next to last occasion. A cost  $C$  is

incurred, and the total expected expense is this cost plus the expected expense for the last occasion. Generalizing to the  $k \geq 2$  case simply involves substituting  $E_C(k - 1)$  in place of  $E_C(1)$  in (5) and yields

$$E_C(k) = \min[(1 - p_\theta)E_C(k - 1) + p_\theta L, C + E_C(k - 1)], \tag{6}$$

$k = 2, 3, \dots, n$ . We note that, with the convention that  $E_C(0) = 0$ , (6) holds for the case of  $k = 1$  as well. Hence, by means of the recurrence relation (6),  $E_C(k)$  can be computed iteratively for  $k = 1, 2, \dots, n$ .

At each occasion of the decision-making process, it is of interest to record the action that minimizes the rhs of (6). A rule that specifies the optimal action as a function of the available information is called the *optimal policy*. It is shown in Appendix A that the optimal policy, given only climatological information, is of the following form:

- (i) do not protect on the first  $n - k_C$  occasions;
- (ii) protect on the last  $k_C$  occasions;

for some  $k_C$ ,  $0 \leq k_C \leq n$ . The exact number of occasions  $k_C$  on which it is optimal to protect depends on the parameters  $C/L$  and  $p_\theta$  of the decision-making process (see Appendix A). The structure of the optimal policy has the intuitive explanation that, if an individual can only afford to protect on  $k_C$  occasions, then it is better to postpone protecting as long as possible within the  $n$ -occasion time period, avoiding unnecessary protection in the event that a loss  $L$  is incurred.

*b. Perfect information*

We let  $E_P(k)$  denote the minimum total expected expense for the last  $k$  occasions of a  $n$ -occasion time period ( $k = 1, 2, \dots, n$ ) when perfect information is available and given that no loss  $L$  has yet been incurred. For the  $k = 1$  case (i.e., the static cost-loss ratio situation),

$$E_P(1) = p_\theta C, \tag{7}$$

since 1) protection is always optimal for  $C < L$  when the perfect forecast indicates the occurrence of adverse weather and 2) adverse weather occurs with probability  $p_\theta$ . Then for the  $k = 2$  case, using the fact that the  $k = 1$  case has already been solved with  $E_P(1)$  given by (7),

$$E_P(2) = (1 - p_\theta)E_P(1) + p_\theta \min[L, C + E_P(1)]. \tag{8}$$

The first term on the rhs of (8) represents the total expected expense over the last two occasions given that adverse weather will not occur on the next to last occasion (in which case it is, of course, optimal not to protect), weighted by the probability  $1 - p_\theta$  of no adverse weather occurring. The second term rep-

resents the corresponding expense given that adverse weather will occur on the next to last occasion, weighted by the probability  $p_\theta$  of adverse weather occurring, with the two terms inside the minimum representing the expenses for not protecting and protecting, respectively. Generalizing to the  $k$ -occasion case simply involves substituting  $E_F(k - 1)$  in place of  $E_F(1)$  in (8) and yields

$$E_F(k) = (1 - p_\theta)E_F(k - 1) + p_\theta \min[L, C + E_F(k - 1)], \quad (9)$$

$k = 1, 2, \dots, n$ , with the convention that  $E_F(0) = 0$ . Hence, by means of the recurrence relation (9),  $E_F(k)$  can be computed iteratively for  $k = 1, 2, \dots, n$ .

It is shown in Appendix A that the optimal policy, given perfect information, is of the following form:

- (i) do not protect on the first  $n - k_P$  occasions;
- (ii) protect on the last  $k_P$  occasions whenever  $\theta = 1$ ;

for some  $k_P$ ,  $1 \leq k_P \leq n$ . The exact number of occasions  $k_P$  on which it is optimal to protect, whenever  $\theta = 1$ , depends on the parameters  $C/L$  and  $p_\theta$  (see Appendix A). This form of optimal policy is the same as that for climatological information, except that  $k_C \leq k_P$  (see Appendix A); that is, protection is optimal at least as early in the time period when perfect information is available, because protection is needed only on those occasions on which adverse weather will occur.

*c. Imperfect forecasts*

We let  $E_F(k)$  denote the minimum total expected expense for the last  $k$  occasions of a  $n$ -occasion time period ( $k = 1, 2, \dots, n$ ) when imperfect forecasts are available and given that no loss  $L$  has yet been incurred. For the  $k = 1$  case (i.e., the static cost-loss ratio situation),

$$E_F(1) = (1 - p_Z) \min(p_0 L, C) + p_Z \min(p_1 L, C). \quad (10)$$

Here  $E_F(1)$  is similar to the corresponding expression (4) for climatological information, except that two terms appear instead of one. These two terms arise because of the two different conditional probabilities of occurrence of adverse weather  $p_0$  and  $p_1$ , depending on whether or not adverse weather is forecast. The first term is weighted by the probability  $1 - p_Z$  of a forecast of no adverse weather and the second term is weighted by the probability  $p_Z$  of a forecast of adverse weather.

Then for the  $k = 2$  case, using the fact that the  $k = 1$  case already has been solved with  $E_F(1)$  given by (10),

$$E_F(2) = (1 - p_Z) \min[(1 - p_0)E_F(1) + p_0 L, C + E_F(1)] + p_Z \min[(1 - p_1)E_F(1) + p_1 L, C + E_F(1)]. \quad (11)$$

The first term on the rhs of (11) represents the minimum total expected expense over the last two occasions when no adverse weather is forecast on the next to last occasion (i.e.,  $Z = 0$ ), whereas the second term represents the corresponding expense for the case of a forecast of adverse weather on the next to last occasion (i.e.,  $Z = 1$ ). The two terms are weighted in the same manner as in (10). Generalizing to the  $k$ -occasion case simply involves substituting  $E_F(k - 1)$  in place of  $E_F(1)$  in (11) and yields

$$E_F(k) = (1 - p_Z) \min[(1 - p_0)E_F(k - 1) + p_0 L, C + E_F(k - 1)] + p_Z \min[(1 - p_1)E_F(k - 1) + p_1 L, C + E_F(k - 1)], \quad (12)$$

$k = 1, 2, \dots, n$ , with the convention that  $E_F(0) = 0$ . Hence, by means of the recurrence relation (12),  $E_F(k)$  can be computed iteratively for  $k = 1, 2, \dots, n$ .

It is shown in Appendix A that the optimal policy, given imperfect forecasts, is of the following form:

- (i) do not protect on the first  $n - k'_F$  occasions;
- (ii) protect on occasions  $n - k'_F + 1$  through  $n - k'_F$  whenever  $Z = 1$ ;
- (iii) protect on the last  $k'_F$  occasions;

for some  $k_F$  and  $k'_F$ ,  $0 \leq k'_F \leq k_F \leq n$  ( $k'_F \leq k_F$  because  $p_0 \leq p_1$ ; see Appendix A). The exact values of  $k_F$  and  $k'_F$  depend on the parameters  $C/L$ ,  $p_\theta$ ,  $p_0$ , and  $p_1$  (see Appendix A).

This form of optimal policy is somewhat similar to a combination of the optimal policies for climatological information and perfect information. As in the case of climatological information, for the last  $k'_F$  occasions it is always optimal to protect. As in the case of perfect information, it is optimal to protect whenever adverse weather is forecast on the last  $k_F$  occasions. Figure 2 presents a comparison of the forms of optimal policies for these three types of weather information. It is shown in Appendix A that the four quantities that specify the different optimal policies satisfy the ordering

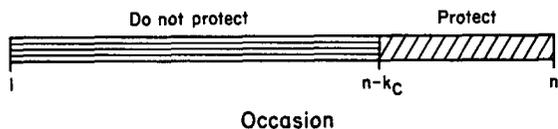
$$k'_F \leq k_C \leq k_F \leq k_P. \quad (13)$$

Thus, Fig. 2 illustrates an ordering among the three forms of optimal policies that must always hold.

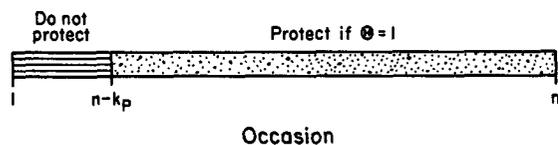
*d. Quality/value relationships*

One of the primary purposes of studying the dynamic cost-loss ratio decision-making situation is to investigate the nature of the relationship between the "quality" (i.e., some measure of the correspondence

(a) Climatological information



(b) Perfect information



(c) Imperfect forecasts

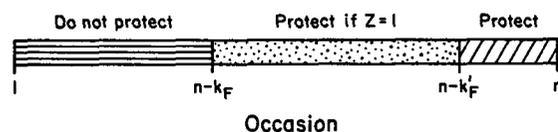


FIG. 2. Structure of optimal policies for (a) climatological information, (b) perfect information, and (c) imperfect information.

between forecasts and observations of adverse weather/ no adverse weather) and the “value” of forecasts. We now describe some general characteristics of quality/value relationships that will always hold. In Section 4, empirical quality/value curves are calculated for specific numerical values of the parameters; namely, the cost-loss ratio  $C/L$ , the climatological probability  $p_\theta$  of adverse weather, and the number of occasions  $n$  in the time period of the model.

Imperfect forecasts of the form considered in this paper are characterized by the two conditional probabilities of adverse weather,  $p_0$  and  $p_1$ , depending on whether or not adverse weather is forecast. In dealing with such forecasts, it is convenient to make one additional assumption; namely, that  $p_Z = p_\theta$ . That is, adverse weather is assumed to be forecast with the same long-run frequency as its long-run frequency of occurrence. This condition can be thought of as a kind of “reliability in the large.” With the imposition of the requirement that  $p_Z = p_\theta$ , the two conditional probabilities are related by

$$p_0 = (1 - p_1)[p_\theta / (1 - p_\theta)] \tag{14}$$

[see (1)]. Thus the imperfect forecasts can be characterized by the single parameter  $p_1$ , in addition to the climatological, and in this case predictive, probability  $p_\theta$ .

A natural ordering among the values of the different types of weather information holds for the dynamic cost-loss ratio decision-making model. Specifically, as shown in Katz *et al.* (1981) for the case of a more

general Markov decision process, the total expected expenses satisfy the ordering

$$E_P(n) \leq E_F(n) \leq E_C(n). \tag{15}$$

In terms of the concept of the value of information as measured relative to climatological information, (15) implies that the value of imperfect forecasts satisfies

$$0 \leq V_P(n) \leq V_F(n). \tag{16}$$

The inequality (16) means that, on the one hand, imperfect forecasts cannot be of negative value (ignoring the costs, if any, of preparing and/or acquiring the forecasts) and, on the other hand, imperfect forecasts cannot be any more valuable than perfect information. This result is essentially a special case of a fundamental theorem of Blackwell (1953) concerning the comparative value of experiments. White (1966) also has shown that the value of imperfect forecasts is always nonnegative in the case of an “infinite horizon” problem (i.e., letting the number of occasions  $n$  in the time period tend to infinity) for a Markov decision process. Hilton (1981) reviews currently existing results concerning the value of information in a more general decision-theoretic setting.

We mention one further result concerning quality/value relationships for the dynamic cost-loss ratio decision-making model. Under the assumption that  $p_Z = p_\theta$ , any measure of the quality of forecasts is simply a function of the conditional probability  $p_1$  of adverse weather given a forecast of adverse weather. Here quality is necessarily an increasing function of  $p_1$ , since  $p_1$  ranges from  $p_\theta$  for climatological information to one for perfect information. It can be shown that

$$\frac{\partial E_F(n)}{\partial p_1} \leq 0, \tag{17}$$

or equivalently,

$$\frac{\partial V_F(n)}{\partial p_1} \geq 0. \tag{18}$$

That is, the value of the forecasts cannot decrease as the quality increases, which is again a special case of a theorem of Blackwell (1953). On the other hand, (18) does not imply anything about the nature of the quality/value relationships for the dynamic cost-loss ratio model, except that value is necessarily an increasing function of quality. The exact quality/value relationships for the  $n$ -occasion time period problem are quite complex nonlinear functions and they are difficult to derive analytically.

However, in the case of a static cost-loss ratio decision-making model (i.e.,  $n = 1$ ), a quality/value relationship can be derived analytically (see Appendix B). As illustrated in Fig. 3, even in this nondynamic

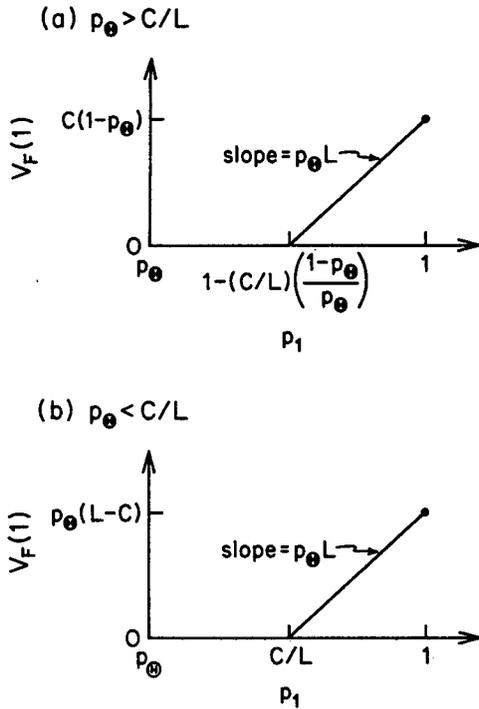


FIG. 3. Value of imperfect forecasts in static cost-loss ratio situation,  $V_F(1)$ , as a function of  $p_1$  for (a)  $p_0 > C/L$  and (b)  $p_0 < C/L$ .

case the quality/value relationship is nonlinear, with the value of imperfect forecasts being zero for  $p_1$  between  $p_0$  and a certain threshold,  $p_1^*$  say, and then increasing linearly for  $p_1$  between  $p_1^*$  and one. Specifically, it is shown in Appendix B that

$$V_F(1) = \begin{cases} 0, & \text{if } p_0 < p_1 < p_1^*, \\ p_0 L(p_1 - p_1^*), & \text{if } p_1^* < p_1 < 1, \end{cases} \quad (19)$$

where

$$p_1^* = \begin{cases} C/L, & \text{if } p_0 < C/L, \\ 1 - (C/L)[(1 - p_0)/p_0], & \text{if } p_0 > C/L. \end{cases} \quad (20)$$

#### 4. Numerical results

In this section we present some numerical results derived from the  $n$ -occasion time period dynamic model of the cost-loss ratio situation. Two specific cases are used to illustrate the results; 1) Case A:  $p_0 = p_Z = 0.2$  and  $C/L = 0.3$ , and 2) Case B:  $p_0 = p_Z = 0.4$  and  $C/L = 0.3$ .<sup>2</sup> Note that  $p_0 < C/L$  in Case A, whereas  $p_0 > C/L$  in Case B. The results presented here relate primarily to optimal policies and the value of different types of information.

<sup>2</sup> Without loss of generality, we can take  $L = 1$ . This choice of  $L$  is equivalent to translating total expected expense (value) into total expected expense (value) per unit loss. This transformation is employed in presenting the results in Figs. 5 and 6.

#### a. Optimal policies

Optimal policies for Cases A and B are depicted in Figs. 4a and 4b, respectively, for a  $n = 16$  occasion time horizon. In Case A (Fig. 4a), protection is never taken for climatological information (i.e.,  $k_C = 0$  for  $p_1 = 0.2$ ), whereas protection is taken when  $Z = 1$  on the last 12 occasions of the time period for perfect information (i.e.,  $k_P = 12$  for  $p_1 = 1.0$ ). For imperfect forecasts (i.e., for  $0.2 < p_1 < 1.0$ ), the number of occasions on which protection is taken when  $Z = 1$  (i.e.,  $k_F$ ) increases as  $p_1$  increases (i.e., as the quality of the imperfect forecasts improves). Note that it is never optimal to protect when  $Z = 0$  in Case A (i.e.,  $k'_F = 0$ ). These results are, of course, consistent with the relationships between  $k_C$ ,  $k_P$ ,  $k_F$ , and  $k'_F$  described in Section 3c [see (13)].

For Case B, the optimal policy as a function of  $p_1$  is depicted in Fig. 4b. In this case, protection is taken on the last occasion of the time period for climatological information ( $k_C = 1$ ), whereas protection is taken when  $Z = 1$  on the last 6 occasions of the time period for perfect information ( $k_P = 6$ ). As  $p_1$  increases, the number of occasions on which protective action

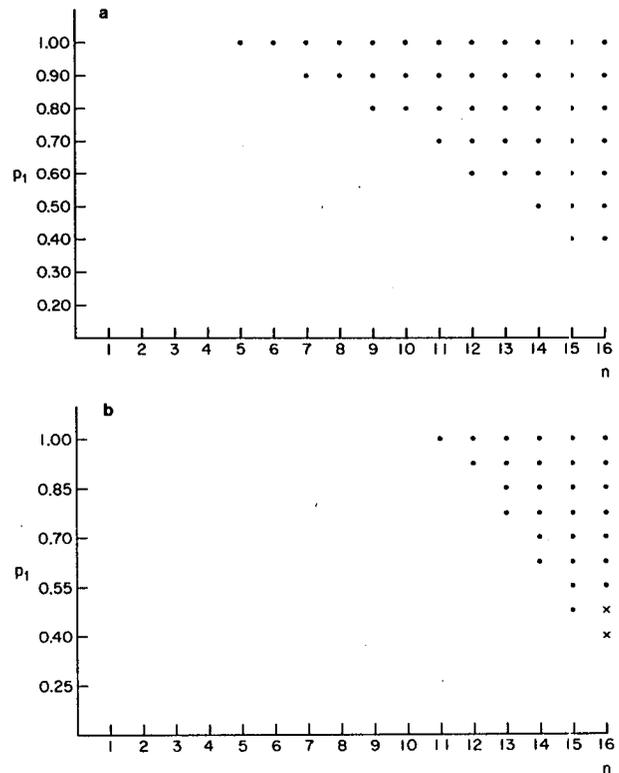


FIG. 4. Optimal policies for  $n = 16$  time horizon for various values of  $p_1$ . (a) Case A:  $p_0 = p_Z = 0.2$  and  $C/L = 0.3$ . (b) Case B:  $p_0 = p_Z = 0.4$  and  $C/L = 0.3$ . A dot denotes "protect when  $Z = 1$ " and a cross denotes "protect when  $Z = 1$  or  $Z = 0$ ." The absence of a symbol indicates that the optimal action is "do not protect."

is taken when  $Z = 1$  remains constant or increases, as expected. Note that when  $p_1 < 0.55$  protective action should be taken on the last occasion of the time period irrespective of whether  $Z = 1$  or  $Z = 0$  (because  $p_0 > C/L = 0.3$  when  $p_1 < 0.55$ ). Thus, for forecasts of low quality (i.e., low values of  $p_1$ ), the probability of adverse weather following a forecast of no adverse weather is still high enough to justify taking protective action.

In a comparison of the optimal policies in Cases A and B (cf. Figs. 4a and 4b), it can be seen that protective action begins later in the time period for Case B than for Case A. Specifically, when  $Z = 1$ , protective action is taken on approximately one-half as many occasions in Case B as it is in Case A (for a given value of  $p_1$ ). However, because of the higher value of  $p_0 = p_Z$  in Case B, adverse weather is expected to occur more frequently and is forecast (i.e.,  $Z = 1$ ) on twice as many occasions in Case B as in Case A. Thus, protective action will be taken on approximately the same number of occasions in both cases, and the expected portion of the total expense associated with taking protective action is therefore approximately the same in the two cases.

b. Value of forecasts

1) QUALITY/VALUE RELATIONSHIPS

The value of imperfect forecasts  $V_F(n)$  as a function of  $p_1$  for Cases A and B is depicted in Figs. 5a and 5b, respectively, for  $n = 1, 2, 5, 10$  and  $16$ . Since  $p_1$  represents an indicator of the quality of the forecasts (i.e., quality increases as  $p_1$  increases), the curves in Fig. 5 describe the quality/value relationships in the respective cases for the indicated time horizons. In particular, it should be noted that  $V_F(n) = V_P(n)$  when  $p_1 = 1.0$ .

First, as mentioned in Section 3d,  $V_F(n)$  remains constant or increases as  $p_1$  increases in both cases. Moreover,  $V_F(n)$  is a nonlinear function of  $p_1$  for all time horizons, including the static ( $n = 1$ ) situation. With regard to the latter,  $V_F(1)$  consists of two line segments in both cases [cf. (19) and Fig. 3]. In Case A(B),  $V_F(1) = 0$  for  $p_1 \leq 0.3(0.55)$  and then  $V_F(1)$  increases linearly for  $p_1 > 0.3(0.55)$  to  $V_F(1) = 0.14(0.18)$  when  $p_1 = 1$ . When  $n > 1$ , the relationship between  $V_F(n)$  and  $p_1$  is obviously nonlinear. Overall, these quality/value curves exhibit the same general characteristics in both Case A and Case B.

It is important to recognize that situations involving different time horizons (i.e., different values of  $n$ ) represent different decision-making problems, in the same sense that different values of the cost-loss ratio  $C/L$  represent different decision makers. Thus, comparisons of  $V_F(n)$  as a function of  $p_1$  for different values of the time horizon  $n$  must be interpreted with some care. Notwithstanding this caveat, it is of some interest to compare the quality/value curves in

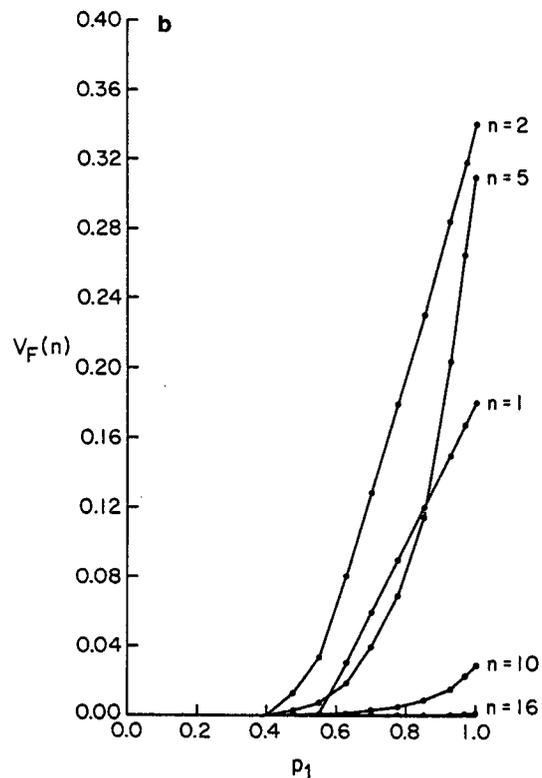
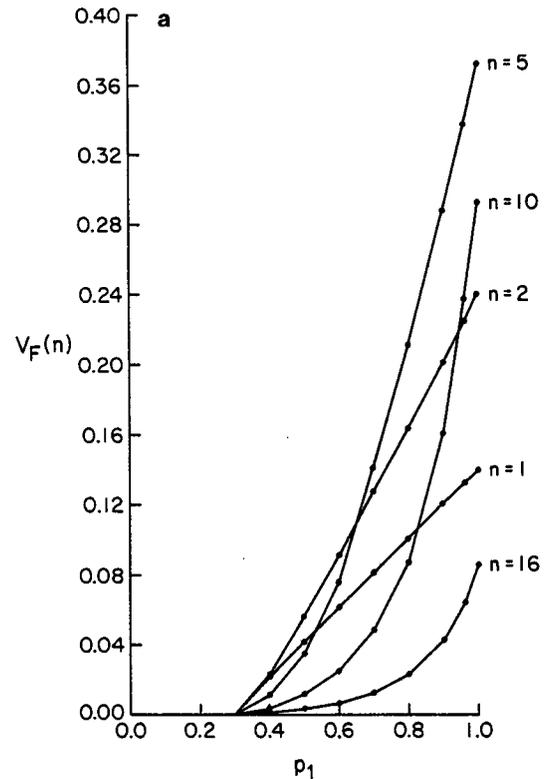


FIG. 5. The value of imperfect forecasts  $V_F(n)$  as a function of  $p_1$  for time horizons  $n = 1, 2, 5, 10$  and  $16$ . (a) Case A:  $p_0 = p_Z = 0.2$  and  $C/L = 0.3$ . (b) Case B:  $p_0 = p_Z = 0.4$  and  $C/L = 0.3$ .

Fig. 5 for different values of the time horizon  $n$ . For large values of  $p_1$  (i.e., for relatively high quality forecasts),  $V_F(n)$  increases as  $n$  increases for small values of  $n$  (i.e., for  $n \leq 5$  in Case A and for  $n \leq 2$  in Case B) and then  $V_F(n)$  decreases for larger values of  $n$  (i.e., for  $n > 5$  in Case A and for  $n > 2$  in Case B). This initial increase in value results from substantial differences between the optimal policies dictated by the forecasts and the optimal policies dictated by climatological information ( $p_1 = p_\theta$ ) for such values of  $n$  [the differences in optimal policies are less pronounced in the static ( $n = 1$ ) situation]. As the time horizon  $n$  increases still further for these values of  $p_1$ , the optimal policy based on the forecasts necessarily involves taking protective action on a smaller percentage of the occasions (otherwise the total expected expense associated with taking protective action would become undesirably large), and this policy corresponds more closely to that associated with climatological information.

For smaller values of  $p_1$  (i.e., for forecasts of lower quality), the optimal policy associated with the forecasts approaches the optimal policy associated with climatological information even more rapidly (than it does for larger values of  $p_1$ ) as the time horizon  $n$  increases, with the result that the "changeover" from increasing  $V_F(n)$  to decreasing  $V_F(n)$  occurs for smaller values of  $n$ . In the limit as  $n$  becomes large, the optimal policy based on imperfect forecasts approaches that based on climatological information, and the value of the forecasts approaches zero. This result can be explained heuristically by recognizing that the total expected expense for all types of information approaches  $L$  as  $n$  becomes large (or, equivalently, that the probability of suffering the loss  $L$  approaches one as  $n$  becomes large, regardless of the quality of the information).

Comparison of the quality/value curves for Cases A ( $p_\theta < C/L$ ) and B ( $p_\theta > C/L$ ) reveals some overall similarities and some specific differences. With regard to the latter, it is of interest to note that, irrespective of the length of the time horizon  $n$ , the value of the forecasts  $V_F(n)$  in Case A is zero for  $p_1 \leq 0.30$ . In Case B, on the other hand,  $V_F(n) = 0$  for  $p_1 \leq 0.55$  when  $n = 1$ , whereas  $V_F(n) > 0$  for  $p_1 > 0.40$  for larger values of  $n$ . That is, in this latter case forecasts of relatively low quality are of some value in situations involving longer time horizons, whereas such forecasts are of no value in the static ( $n = 1$ ) situation.

## 2) VALUE VERSUS COST-LOSS RATIO

The value of imperfect forecasts  $V_F(n)$  as a function of the cost-loss ratio  $C/L$  for Case A is depicted in Fig. 6 for selected values of forecast quality  $p_1$  and time horizon  $n$ . In the static  $n = 1$  situation (Fig. 6a), the curves for the various values of  $p_1$  each consist of line segments with positive and negative

slopes, respectively. Specifically,  $V_F(1)$  increases linearly from  $C/L = 1 - p_1$  to  $C/L = 0.2$  and then  $V_F(1)$  decreases linearly from  $C/L = 0.2$  to  $C/L = p_1$ . Thus, the maximum value of  $V_F(1)$  is attained at  $C/L = p_\theta = 0.2$ . In the static situation, then, imperfect forecasts are of greatest value for decision makers whose cost-loss ratios are equal to the climatological probability of adverse weather. For longer time horizons, the curves (Figs. 6b, c, and d) indicate that the maximum value of  $V_F(n)$  shifts toward smaller values of  $C/L$  as  $n$  increases. For example, the maximum value of  $V_F(10)$  occurs near  $C/L = 0.10$ . Thus, as  $n$  increases, imperfect forecasts are of greater value for decision makers with cost-loss ratios somewhat less than  $p_\theta$  than for decision makers with cost-loss ratios equal to  $p_\theta$ .

It is also of interest to note in Fig. 6 that the "sensitivity" of  $V_F(n)$  to changes in forecast quality increases as the time horizon increases in length for a broad range of values of the cost-loss ratio. That is, the differences between the curves for two particular values of  $p_1$  increase as  $n$  increases. In absolute terms, then, improvements in quality will lead to larger increases in value for some decision makers (i.e., for some values of  $C/L$ ) when  $n$  is large than when  $n$  is small (however, recall that the value of all forecasts approaches zero as  $n$  becomes still larger). On the other hand, the shift in these curves (Fig. 6) toward lower values of  $C/L$  as  $n$  increases indicates that the value of the forecasts to decision makers with large values of  $C/L$  decreases quite rapidly as  $n$  increases. Examination of these relationships for Case B yields qualitatively similar results, and these figures were omitted to conserve space.

## 5. Discussion and conclusion

This paper has described a dynamic model for repetitive decision-making in the cost-loss ratio situation. As such, the model represents a potentially useful generalization of the familiar static model of this situation. However, many other extensions of both the static and dynamic models are possible. With regard to the former, Epstein (1969) has described a static model for a  $N$ -action,  $N$ -event generalization of the standard cost-loss ratio situation, and the value of weather forecasts within the framework of this model has recently been investigated by Murphy (1985a). Here, we briefly discuss several possible extensions of the dynamic model formulated in this paper.

One extension of this model would involve changing the nature of the dynamics by assuming a different structure for the expenses within the framework of the two-action, two-event situation. For example, instead of a single complete loss when protective action is not taken and adverse weather occurs, the expense incurred under these conditions could be

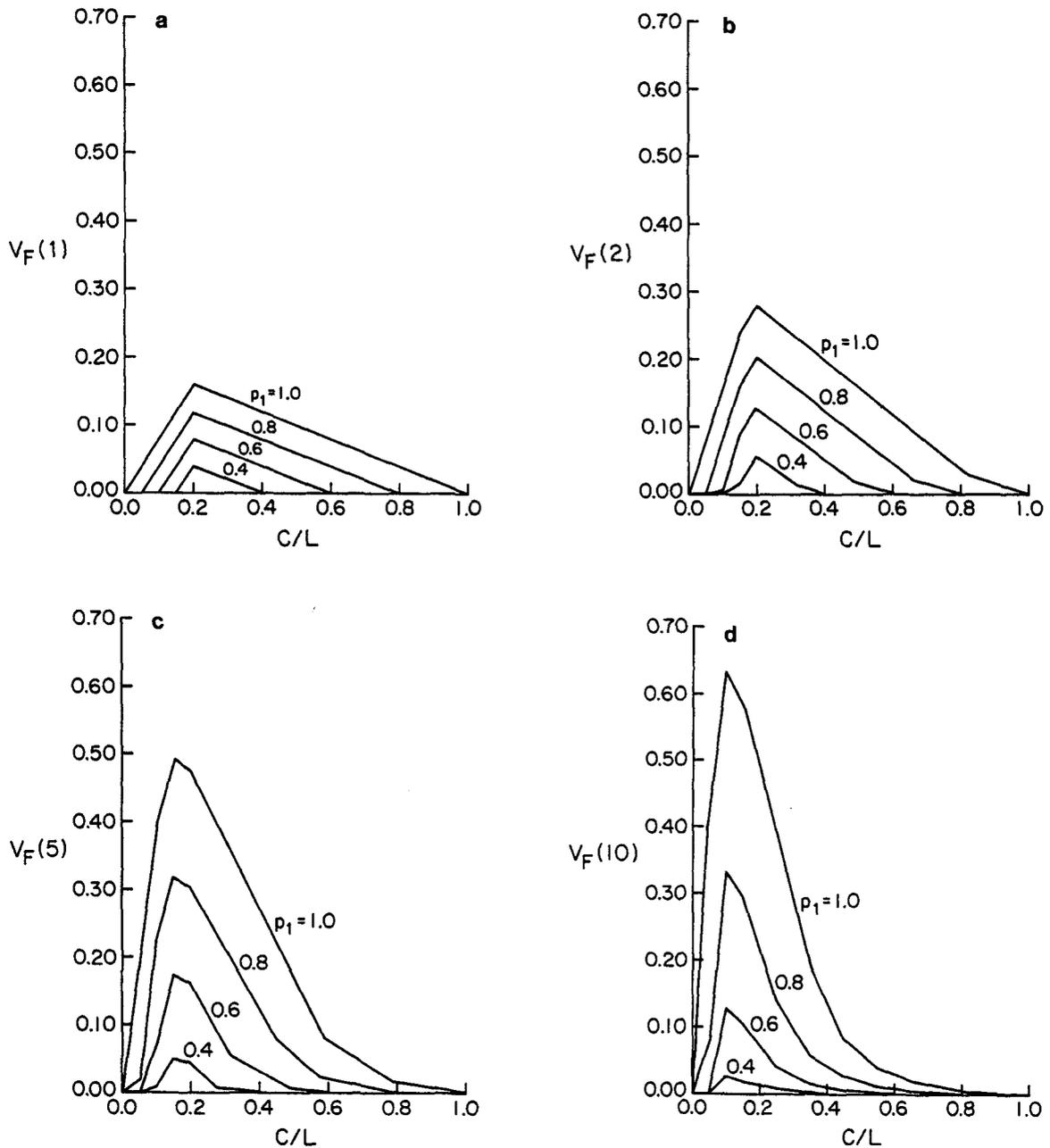


FIG. 6. The value of imperfect forecasts  $V_F(n)$  as a function of  $C/L$  for selected values of  $p_1$  for Case A:  $p_\theta = p_z = 0.2$ . (a)  $V_F(1)$ , (b)  $V_F(2)$ , (c)  $V_F(5)$ , and (d)  $V_F(10)$ .

considered to be only a partial loss. Alternatively, the loss might be treated as recoverable in part over time rather than as complete and final. Another possible extension would consist of employing decision-making criteria other than that of minimizing total expected expense. For instance, one such criterion might prescribe that the decision maker choose actions in such a way as to postpone the occurrence of a (complete) loss as long as possible. In a related vein, it is frequently appropriate in sequential decision-making

problems to discount future expenses (or returns). Thus, it would be of interest to compare the optimal policies and value-of-information estimates obtained when such a discount factor is included in the dynamic model with the results presented here for the nondiscounted case. Moreover, in situations involving large or catastrophic losses, it generally is desirable to take the decision maker's attitude toward risk into account in determining optimal policies and the value of information. Thus, maximizing expected utility might

be a more appropriate decision criterion in this context than minimizing expected expense.

Another type of extension of the model considered in this paper would consist of the formulation of a dynamic model for a decision-making problem involving more than two actions and/or events. For example, various levels of protection and degrees of adverse weather might be considered. In this regard, it would be of interest to formulate a dynamic model for the  $N$ -action,  $N$ -event generalization of the cost-loss ratio situation originally described by Epstein (1969). Finally, it would be desirable to investigate the optimal policies and value of information for more general forms of imperfect forecasts than the calibrated categorical or "primitive" probability forecasts considered here (see Section 2b). Examples of these more realistic imperfect forecasts include the objective and subjective probability of precipitation forecasts currently produced on an operational basis by the National Weather Service (e.g., see Murphy, 1985b).

In conclusion, it should be recalled that the static cost-loss ratio model, despite its obvious simplicity, has served as a useful prototype of many "one-shot" weather-information-sensitive decision-making problems. We believe that the dynamic cost-loss ratio model described in this paper may play a similar role for repetitive decision-making problems. Moreover, it should be noted that the static model is simply a special case (the one-occasion time period) of the dynamic model. Clearly, it would be desirable to investigate this dynamic model further by applying it to specific real-world decision-making problems in which weather forecasts could be used to make more effective and efficient decisions.

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#### APPENDIX A

### Structure of Optimal Policies

#### 1. Climatological information

From (6), it is optimal to protect on the  $k$ th occasion from the end of the time period if

$$C + E_C(k-1) < (1 - p_0)E_C(k-1) + p_0L;$$

that is, if

$$C < p_0[L - E_C(k-1)]. \quad (A1)$$

It can be shown that  $E_C(k)$  is a nondecreasing function of  $k$ , with an asymptotic limit of  $L$ . Thus, a value of  $k$  exists,  $k_C$  say, such that

$$C < p_0[L - E_C(k-1)] \quad \text{for } k \leq k_C,$$

$$C > p_0[L - E_C(k-1)] \quad \text{for } k_C < k \leq n,$$

which establishes the structure of the optimal policy with climatological information.

#### 2. Perfect information

From (9), it is optimal to protect on the  $k$ th occasion from the end of the time period if

$$C + E_F(k-1) < L;$$

that is, if

$$C < L - E_F(k-1). \quad (A2)$$

Here  $E_F(k)$  can be shown to be a nondecreasing function of  $k$ , with an asymptotic limit of  $L$ . Thus, a value of  $k$  exists,  $k_P$  say, such that

$$C < L - E_F(k-1) \quad \text{for } k \leq k_P,$$

$$C > L - E_F(k-1) \quad \text{for } k_P < k \leq n,$$

which establishes the structure of the optimal policy with perfect information.

#### 3. Imperfect forecasts

##### a. Given $Z = 1$

From (12), it is optimal to protect on the  $k$ th occasion from the end of the time period if

$$C + E_F(k-1) < (1 - p_1)E_F(k-1) + p_1L;$$

that is, if

$$C < p_1[L - E_F(k-1)]. \quad (A3)$$

$E_F(k)$  can be shown to be a nondecreasing function of  $k$ , with an asymptotic limit of  $L$ . Thus, a value of  $k$  exists, say  $k_F$ , such that the optimal policy is of the form:

(i) do not protect on the first  $n - k_F$  occasions whenever  $Z = 1$ ;

(ii) protect on the last  $k_F$  occasions whenever  $Z = 1$ .

##### b. Given $Z = 0$

From (12), it is optimal to protect on the  $k$ th occasion from the end of the time period if

$$C + E_F(k-1) < (1 - p_0)E_F(k-1) + p_0L;$$

that is, if

$$C < p_0[L - E_F(k-1)]. \quad (A4)$$

Thus, a value of  $k$  exists,  $k'_F$  say, such that the optimal policy is of the form:

(i) do not protect on the first  $n - k'_F$  occasions whenever  $Z = 0$ ;

(ii) protect on the last  $k'_F$  occasions whenever  $Z = 0$ .

Since  $p_0 \leq p_1$ , it follows that

$$p_0[L - E_F(k-1)] \leq p_1[L - E_F(k-1)].$$

Thus (A3) and (A4) imply that  $k'_F \leq k_F$ , which establishes the structure of the optimal policy with imperfect forecasts.

#### 4. Relationships among optimal policies

Since  $p_\theta \leq p_1 \leq 1$  and  $E_F(k) \leq E_C(k) \leq E_C(k)$ , it follows that

$$p_\theta[L - E_C(k - 1)] \leq p_1[L - E_F(k - 1)] \leq L - E_F(k - 1).$$

Thus (A1), (A2), and (A3) imply that  $k_C \leq k_F \leq k_P$ . Since (6) and (12) imply that  $E_C(k) = E_F(k) = kC$  for  $k \leq \min(k_C, k'_F)$  and since  $p_0 \leq p_\theta$ , it follows that

$$p_0[L - E_F(k - 1)] \leq p_\theta[L - E_C(k - 1)], \quad \text{for } k - 1 \leq \min(k_C, k'_F).$$

Thus (A1) and (A4) imply that  $k'_F \leq k_C$ , which establishes (13).

#### APPENDIX B

##### Quality/Value Relationship for Static Cost-Loss Ratio Situation

In this situation, we are given that  $n = 1$  and  $p_Z = p_\theta$  [i.e., that (14) holds].

Case 1:  $p_\theta > C/L$ . Using (14), (10) can be expressed as

$$E_F(1) = \begin{cases} C, & \text{if } p_\theta < p_1 < 1 - (C/L)[(1 - p_\theta)/p_\theta] \\ (1 - p_1)p_\theta L + p_\theta C, & \\ \text{if } 1 - (C/L)[(1 - p_\theta)/p_\theta] < p_1 < 1. \end{cases} \quad (\text{B1})$$

Then, applying (2), (4), and (B1), the value of imperfect forecasts is given by

$$V_F(1) = \begin{cases} 0, & \text{if } p_\theta < p_1 < 1 - (C/L)[(1 - p_\theta)/p_\theta] \\ C(1 - p_\theta) - (1 - p_1)p_\theta L, & \\ \text{if } 1 - (C/L)[(1 - p_\theta)/p_\theta] < p_1 < 1. \end{cases} \quad (\text{B2})$$

Case 2:  $p_\theta < C/L$ . Using (14), (10) can be expressed as

$$E_F(1) = \begin{cases} (1 - p_1)p_\theta L + p_\theta C, & \text{if } C/L < p_1 < 1, \\ p_\theta L, & \text{if } p_\theta < p_1 < C/L. \end{cases} \quad (\text{B3})$$

Then, applying (2), (4), and (B3), the value of imperfect forecasts is given by

$$V_F(1) = \begin{cases} p_\theta(p_1 L - C), & \text{if } C/L < p_1 < 1, \\ 0, & \text{if } p_\theta < p_1 < C/L. \end{cases} \quad (\text{B4})$$

Finally, we note that both (B2) and (B4) can be expressed in the general form (19).

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